ENVIRONMENTAL AND SOCIO ECONOMIC ASPECTS

OPTIMISATION OF FILLET SIZE IN A RECTANGULAR DUCT AND A RECTANGULAR OPEN CHANNEL

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ABSTRACT

Optimum size of fillets at the corners of a rectangular duct or a rectangular open channel carrying water to minimize the hydraulic head loss has been worked out. For all the cases viz. a rectangular open channel or a rectangular duct running full or not running full, the optimum size has been found to be 0.24R where R is the hydraulic mean radius.

INTRODUCTION

In rectangular ducts and open channels, usually fillets are provided. The main objective of fillets is to reduce concentration of stresses at the corners. An additional advantage is that the design bending moments are reduced to some extent. However, another aspect of the fillets which is generally ignored while fixing the size of fillets is to reduce the hydraulic head loss in the duct or open channel. This aspect assumes paramount importance in the case of ducts or channels carrying water to or from a hydropower plant as each millimeter of head saved can result in the generation of thousands of more units of energy. In this paper, the optimum size of fillets to minimize the hydraulic head loss has been worked out in the case of an open rectangular channel or a rectangular duct, running full or not running full. Incidentally, in each case, the optimum size of fillet has been found to be equal to 0.24R where R is the hydraulic mean radius.

THEORECTICAL CONCEPT

The velocity(v) of flow for a given discharge(Q) depends on the area of x-section(A). The governing relation is

$$Q = V*A$$

The flow velocity is computed by Manning's formula

$$v = \frac{1}{n} * R^{2/3} * s^{1/2}$$

{where R is Hydraulic Mean Radius (i.e. A/P),

P is the wetted perimeter.

S is the Longitudinal slope

n is rugosity coefficient

Derivation of Mathematical Relation

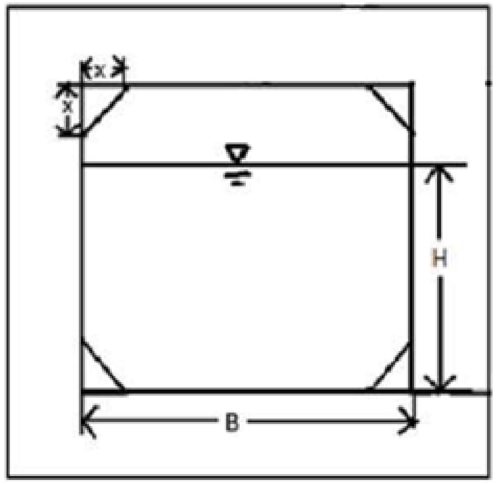
Two cases have been considered and are detailed below i.e.

(i) A Rectangular open channel or a rectangular duct not running full

(ii) A Rectangular duct running full :

CASE I : Size of Fillet in a Rectangular Open Channel or a Rectangular Duct not Running Full

Consider a rectangular duct of width B and depth H with fillet of SIZE x at the corners.



Area (A) of rectangular duct

$A = B*H$

Wetted perimeter(P) of rectangular duct

$P = B+2H$

With provision of fillet,

$A_1 = (B*H)-x^2$

$P_1 = [B+(2*H)]-[2*x*(2-\sqrt{2})]$

$\delta A = x^2$

$\delta P = 2*x*(2-\sqrt{2})$

$= 1.2*x$

Velocity (V) of fluid flowing through the rectangular duct

$V = Q/A$ (Q is discharge of fluid)

By Manning's equation

$V = \frac{1}{n} R^{2/3} s^{1/2}$ {where n= rugosity coefficient

R= Hydraulic Radius

s = longitudinal slope of the duct}

$$s^{1/2} = \frac{nQ}{A} R^{-2/3}$$

$$s = \frac{n^2Q^2}{A^2} R^{-4/3}$$

$$= \frac{kR^{-4/3}}{A^2} \text{(where } k=n^2Q^2 \text{)}$$

$$= \frac{k}{A^{\frac{10}{3}}} * P^{4/3}$$

$= k*P^{4/3}*A^{-10/3}$

$s_1 = \frac{k(P-\delta P)^{4/3}}{(A-\delta A)^{10/3}}$

As δP and δA are small compared to P and A,

$\therefore s_1 = \frac{k\left(P^{4/3}-\frac{4}{3}P^{1/3}\delta P\right)}{\left(A^{10/3}-\frac{10}{3}A^{7/3}\delta A\right)}$

$= k\left(P^{4/3}-\frac{4}{3}P^{1/3}\delta P\right)\left(A^{-10/3}+\frac{10}{3}A^{-13/3}\delta A\right)$

$= kP^{4/3}A^{-10/3}\left(1-\frac{4}{3}\frac{\delta P}{P}+\frac{10}{3}\frac{\delta A}{A}\right)$

$= s\left(1-\frac{4}{3}\frac{\delta P}{P}+\frac{10}{3}\frac{\delta A}{A}\right)$

$\delta s = s - s_1 = s\left(\frac{4}{3}\frac{\delta P}{P}-\frac{10}{3}\frac{\delta A}{A}\right)$

$\frac{\delta s}{s} = \frac{4}{3}\frac{\delta P}{P}-\frac{10}{3}\frac{\delta A}{A}$

For $\frac{\delta s}{s}$ to be maximum, $\frac{dy}{dx} = 0$ where $y = \frac{\delta s}{s}$

$\delta P = 2*x*(2-\sqrt{2})$

$= 1.2*x$

$\delta A = x^2$

$\therefore \frac{\delta s}{s} = y = \frac{4}{3}\left(\frac{1\cdot 2x}{P}\right)-\frac{10}{3}\left(\frac{x^2}{A}\right)$

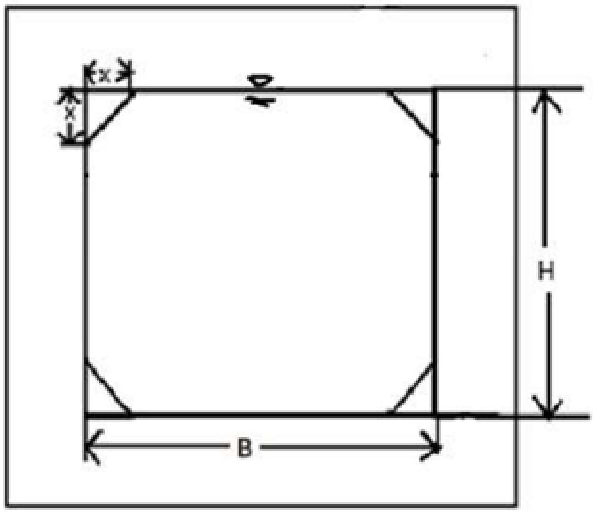
$\frac{dy}{dx} = \frac{1.6}{P}-\frac{20x}{3A} = 0$, or

$\frac{1\cdot 6}{P} = \frac{20x}{3A}$ or $x = \frac{4\cdot 8A}{20P} = 0.24*\frac{A}{P} = 0.24*R$

X=0.24R

CASE II : Rectangular Duct Running Full

Consider a rectangular duct of width B and depth H with fillet x at all the four corners.



Area (A) of rectangular duct

$A = B*H$

Wetted perimeter (P) of rectangular duct

$P = 2*(B+H)$

$A_1 = (B*H)-(2*x^2)$

$P_1 = 2*(B+H)-[4*x*(2-\sqrt{2})$

$\delta A= 2*x^2$

$\delta P= 4*x*(2-\sqrt{2})$

$= 2.4*x$

Velocity (V) of fluid flowing through the rectangular duct

V = Q/A(Q is discharge of fluid)

By Manning's equation

$V = \frac{1}{n} R^{2/3} s^{1/2}$ {where n = manning's coefficient

R = Hydraulic Radius

s = longitudinal slope of the duct}

$$s^{1/2} = \frac{nQ}{A} R^{-2/3}$$

$$s = \frac{n^2Q^2}{A^2} R^{-4/3}$$

$$= \frac{kR^{-4/3}}{A^2} \text{(where } k=n^2Q^2 \text{)}$$

$$= \frac{k}{A^{\frac{10}{3}}} * P^{4/3}$$

$= k*P^{4/3}*A^{-10/3}$

$s_1 = \frac{k(P-\delta P)^{4/3}}{(A-\delta A)^{10/3}}$

As δP and δAare small compared to P and A,

$\therefore s_1 = \frac{k\left(P^{4/3}-\frac{4}{3}P^{1/3}\delta P\right)}{\left(A^{10/3}-\frac{10}{3}A^{7/3}\delta A\right)}$

$= k\left(P^{4/3} - \frac{4}{3}P^{1/3}\delta P\right)\left(A^{-10/3} + \frac{10}{3}A^{-13/3}\delta A\right)$

$= kP^{4/3}A^{-10/3}\left(1 - \frac{4}{3}\frac{\delta P}{P} + \frac{10}{3}\frac{\delta A}{A}\right)$

$= s\left(1 - \frac{4}{3}\frac{\delta P}{P} + \frac{10}{3}\frac{\delta A}{A}\right)$

$\delta s = s - s_1 = s\left(\frac{4}{3}\frac{\delta P}{P} - \frac{10}{3}\frac{\delta A}{A}\right)$

$\frac{\delta s}{s} = \frac{4}{3}\frac{\delta P}{P} - \frac{10}{3}\frac{\delta A}{A}$

For $\frac{\delta s}{s}$ to be maximum, $\frac{dy}{dx} = 0$ where $y = \frac{\delta s}{s}$

δP = 4*x*(2-√2)

= 2.4*x

δA= 2*x^2

$\therefore \frac{\delta s}{s} = y = \frac{4}{3}\left(\frac{2\cdot 4x}{P}\right) - \frac{10}{3}\left(\frac{2x^2}{A}\right)$

$\frac{dy}{dx} = \frac{3.2}{P} - \frac{40}{3A} = 0$

$\therefore \frac{3.2}{P} = \frac{40}{3A}$

$X = \frac{9.6A}{40P} = 0.24*\frac{A}{P}$

= 0.24* R

X = 0.24R

ILLUSTRATION

Assume a rectangular duct in section with dimensions as given in Figure 1.

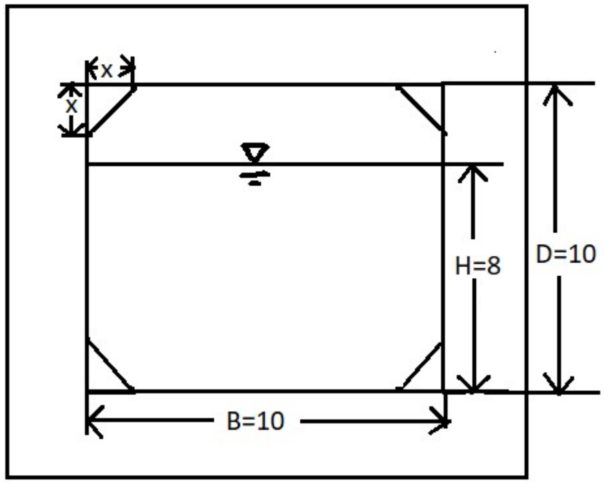


Fig. 1

Using Manning's coefficient (n) = 0.018

Width of the duct (B) = 10m

Depth of running fluid (H) = 8m

Discharge (Q) = 240 m^3/sec

The results obtained for the cases with different values of fillet size has been detailed as below:

FILLET SIZE (x m)		Longitudinal Slope (S)
0*R	0	6.51 x 10^{-4}
0.15*R	0.46	6.39 x 10^{-4}
0.24*R	**0.74**	6.37 x 10^{-4}
0.33*R	1.02	6.39 x 10^{-4}

Assuming a fillet provided in 1km reach of a hydel channel of 10kms length.

For the case of no fillet provided and for Fillet size of duct = 0.24R

Head saved = 1000 x (6.51 x 10^{-4} – 6.37 x 10^{-4}) = 0.014 = 14mm

Extra power generated = 240 x 9.8 x 14/1000 x 1000

= 32.93 kW

Considering load factor = 60%

Extra units of power generated per annum = 32.93 x 0.6 x 24 x 365 = 173080 kWh = 1,73,080 units, say 1.73 Lakh units

Therefore, provision of fillet duct of size 0.24R in 1km reach of hydel channel with 10kms length saved 1.73 Lakh units of power per annum.

CONCLUSION

Instead of keeping the size of fillets in a rectangular open channel or a rectangular duct on an arbitrary/ad-hoc basis, it should be kept equal to 0.24R where R is the hydraulic mean radius of the section; to minimize the hydraulic head loss. This is of paramount importance in the case of channels or ducts carrying water to or from a hydropower plant as each millimeter of head saved can result in the generation of thousands of more units of energy.

Acknowledgement

Warm acknowledgement to Dr. M.R. Goyal, Design Consultant, Shahpurkandi Dam Project, Punjab Water Resources Department for his continuous support and guidance.