APPLYING QUADRATIC IDEAL-COUPLED METHOD IN FREE VIBRATION ANALYSIS OF SEVERAL IRANIAN ARCH DAMS

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ABSTRACT

The finite element formulation of dam-reservoir systems leads to un-symmetric eigenvalue problem including un-symmetric stiffness and mass matrices, when pressure and displacements are considered as the water and the dam unknowns, respectively. The emergence of these matrices causes complexity in analysis of these systems. Recently, the first author developed a new method entitled "quadratic ideal-coupled" to remedy this difficulty [1]. This technique has been applied only into free vibration analysis of Morrow Point arch dam. For more precisely evaluating the capability of this approach, this paper is devoted to take advantage of the quadratic ideal-coupled method in free vibration analysis of some Iranian arch dams.

Keywords *: Arch dam; Decoupled method; Ideal-coupled method; Quadratic ideal-coupled method; Free vibration analysis.*

1. INTRODUCTION

For studying the dynamic behavior of complex systems, such as arch dam-reservoir system, the finite element approach is usually applied. In general, the dynamic behavior of the structures can be studied both in time or frequency domains. In these domains, the analysis may be carried out either by direct or modal method. Consequently, the natural frequencies and corresponding mode shapes are required to be calculated in the dynamic analysis. For this purpose, the eigenvalue problem governing the free vibration of the system should be solved.

The finite element formulation of dam-reservoir systems leads to un-symmetric eigenvalue problem with un-symmetric stiffness and mass matrices, when pressure and displacements are the water and the dam unknowns, correspondingly. For developing a new symmetric version of this problem, Sandberg [2] employed the eigen-vectors of each domain. In this technique, the displacement finite element formulation for the solid and either pressure or displacement potential for the fluid were used. In fact, this scheme was the advent of generating new symmetrizing schemes without employing the coupled mode shapes. Afterwards, Lotfi [3] utilized the decoupled mode shapes instead of the coupled ones in the modal analysis. It is worth mentioning that the decoupled eigen-problems are symmetric. Then, some researchers compared the robustness of the decoupled method with the coupled one [4]. In addition, Aftabi Sani and Lotfi [5] took advantage of new mode shapes named ideal-coupled modes in the modal analysis of concrete arch dams. Actually, these modes were the coupled mode shapes of two ideal fictitious systems. Note that these eigen-problems were symmetric. In recent years, Rezaiee-Pajand et al. proposed a novel strategy entitled "quadratic ideal-coupled method" for finding the eigenpairs of the Marrow Point arch dam [1].

This paper aims to apply the "quadratic ideal-coupled method" for calculating the natural frequencies of three famous Iranian concrete arch dams, namely Karun-3, Karaj and Shahid Rajaiee. For comparison, the results obtained from the decoupled, ideal coupled and true coupled are presented.

2. FREE VIBRATION ANALYSIS

The eigen-problem governing the free vibration of the arch dam-reservoir system has the following shape [6]:

$$\left(\omega^2\begin{bmatrix}\mathbf{M} & \mathbf{0}\\ \mathbf{B} & \mathbf{G}\end{bmatrix}+\begin{bmatrix}-\mathbf{K} & \mathbf{B}^{\mathrm{T}}\\ \mathbf{0} & -\mathbf{H}\end{bmatrix}\right)\begin{bmatrix}\mathbf{r}\\ \mathbf{p}\end{bmatrix}=\begin{bmatrix}\mathbf{0}\\ \mathbf{0}\end{bmatrix} \tag{1}$$

Obviously, this linear eigenvalue problem is analogous to that of the free vibration equation of un-damped systems. Nevertheless, it is not symmetric. For finding the eigenpairs of the dam-reservoir system, this unsymmetrical linear eigenvalue problem should be solved. By directly solving the original eigenvalue problem (1), the actual coupled eigenpairs can be computed. Using the obtained eigenvectors in the modal analysis leads to more accurate responses, in comparison to the other available alternatives. But, the standard eigen-solvers cannot be used for solving this equation because of its un-symmetry. Other researchers indicated that the unsymmetrical eigenvalue solution routines are more time-consuming than symmetrical ones, and from the programming point of view, these tactics are more complicated, as well [4, 5, 7].

3. QUADRATIC IDEAL-COUPLED

Herein, an alternative for the aforementioned eigen-problem is presented [1]. Both the decoupled and ideal-coupled strategy are the special cases of this more general procedure, named "quadratic ideal-coupled method". This strategy is based on two different quadratic eigen-value problems, which are separately proposed in this section.

Using the lower partition equation of Eq. (1) and solving the pressure vector in terms of the displacement vector results in the below relation:

$$\mathbf{p} = \mathrm{w}^2(\mathbf{H}-\mathrm{w}^2\mathbf{G})^{-1}\mathbf{B}\,\mathbf{r} \tag{2}$$

It is clear that $(\mathbf{H}-\mathrm{w}^2\mathbf{G})$ is the subtraction of two matrices, and it should be inverted in the right side of the latter equality. This matrix inversion can be calculated by employing the first-order approximation of the Taylor series as follows [8, 9]:

$$(\mathbf{H}-\mathrm{w}^2\mathbf{G})^{-1} \cong \mathbf{H}^{-1}+\mathrm{w}^2\mathbf{H}^{-1}\mathbf{G}\,\mathbf{H}^{-1} \tag{3}$$

Substituting this relation into the Eq. (2) leads to the coming relation:

$$\mathbf{p} \cong \mathrm{w}^2\left(\mathbf{H}^{-1}+\mathrm{w}^2\mathbf{H}^{-1}\mathbf{G}\,\mathbf{H}^{-1}\right)\mathbf{B}\,\mathbf{r} \tag{4}$$

Inserting the latter relationship into the upper partition equation of Eq. (1) leads to the succeeding result:

$$\left(\mathrm{w}^4\mathbf{Q}\mathbf{G}\mathbf{Q}^{\mathrm{T}}+\mathrm{w}^2(\mathbf{M}+\mathbf{M}_{\mathrm{a}})-\mathbf{K}\right)\mathbf{r}=\mathbf{0} \tag{5}$$

in which

$$\mathbf{Q}=\mathbf{B}^{\mathrm{T}}\mathbf{H}^{-1} \tag{6}$$

$$\mathbf{M}_{\mathrm{a}}=\mathbf{B}^{\mathrm{T}}\mathbf{H}^{-1}\mathbf{B} \tag{7}$$

Clearly, the size of this quadratic eigen-problem is equal to the number of unknown nodal displacements.

At this stage, the second quadratic ideal eigen-problem is established. To achieve this goal, the displacement vector is solved in terms of the pressure vector by using the upper partition equation of Eq. (1). The obtained displacement vector has the next shape:

$$\mathbf{r}=(\mathbf{K}-\mathrm{w}^2\mathbf{M})^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{p} \tag{8}$$

In a similar manner, $(\mathbf{K}-\mathrm{w}^2\mathbf{M})$ can be inverted by applying the first-order approximation of the Taylor series as the subsequent form:

$$(\mathbf{K}-\mathrm{w}^2\mathbf{M})^{-1} \cong \mathbf{K}^{-1}+\mathrm{w}^2\mathbf{K}^{-1}\mathbf{M}\,\mathbf{K}^{-1} \tag{9}$$

Inserting this relation into Eq. (8) leads to the next equality:

$$\mathbf{r} \cong \left(\mathbf{K}^{-1}+\mathrm{w}^2\mathbf{K}^{-1}\mathbf{M}\,\mathbf{K}^{-1}\right)\mathbf{B}^{\mathrm{T}}\mathbf{p} \tag{10}$$

Substitution of the aforesaid relationship into the lower partition of Eq. (1) results in the following equation:

$$\left(\mathrm{w}^4\,\mathbf{S}\,\mathbf{M}\,\mathbf{S}^{\mathrm{T}}+\mathrm{w}^2\left(\mathbf{G}+\mathbf{G}_{\mathrm{a}}\right)-\mathbf{H}\right)\mathbf{p}=\mathbf{0} \tag{11}$$

where

$$\mathbf{S} = \mathbf{B}\,\mathbf{K}^{-1} \tag{12}$$

$$\mathbf{G}_{\mathrm{a}} = \mathbf{B}\,\mathbf{K}^{-1}\mathbf{B}^{\mathrm{T}} \tag{13}$$

The size of this quadratic eigen-problem is equal to the number of the unknown nodal pressures.

Recall that a $n \times n$ quadratic eigen-problem has $2n$ eigen-values. According to the characteristics of the coefficient matrices, the eigen-values may be infinite or finite [10]. And, the finite values may be real or complex. Obviously, the real values are the approximate natural frequencies of the dam-reservoir system, and the other values are fictitious.

The aforesaid two quadratic ideal-coupled eigen-value problems, i.e. Eqs. (5) and (11), can be expressed totally as follows:

$$\left(\mathrm{w}^4 \begin{bmatrix} \mathbf{Q}\,\mathbf{G}\,\mathbf{Q}^{\mathrm{T}} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}\,\mathbf{M}\,\mathbf{S}^{\mathrm{T}} \end{bmatrix} + \mathrm{w}^2 \begin{bmatrix} \mathbf{M} + \mathbf{M}_{\mathrm{a}} & \mathbf{0} \\ \mathbf{0} & \mathbf{G} + \mathbf{G}_{\mathrm{a}} \end{bmatrix} - \begin{bmatrix} \mathbf{K} & \mathbf{0} \\ \mathbf{0} & \mathbf{H} \end{bmatrix} \right) \begin{bmatrix} \mathbf{r} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} \tag{14}$$

The solution of this combined symmetric eigen-problem can be obtained by solving two separate quadratic eigen-value problems. It is worthwhile to mention that omitting the first term of this relation leads to the ideal-coupled problem

Moreover, removing the first term, $\mathbf{M}_{\mathrm{a}}$ and $\mathbf{G}_{\mathrm{a}}$ results in the decoupled problem.

4. SOLVING THE QUADRATIC EIGEN-PROBLEM

For solving the quadratic eigen-problems various alternatives are available. In what follows, some of them are reviewed. Most of the numerical methods dealing directly with the quadratic eigen-problems are the variants of Newton's approaches [10]. Generally, they calculate one eigen-pair at a time. Their rate of convergence is highly dependent on the closeness of the starting guess to the actual solution. Note that, there is no guarantee that the technique will converge to the desired eigen-value even for a suitable initial guess [10].

The classical and most widely used method for finding the solution of the quadratic eigen-problems is linearization, in which a $n \times n$ quadratic eigen-problem can be transformed into a $2n \times 2n$ linear eigen-value problem. Therefore, common linear eigen-solvers incorporated in commercial and noncommercial software packages can be applied. The eigen-values of the quadratic eigen-problem are analogous to its linear form. Also, its eigen-vectors can be obtained from the corresponding linear problem [11].

Additionally, Bathe et al. [12] developed a scheme, named subspace iteration technique, for finding the eigen-pairs of linear problems. This method is appropriate for the finite element model of the huge structures. With the help of this method, any arbitrary number of structural eigenvalues and eigenvectors can be obtained. Rezaiee-Pajand et al. [1] generalized this famous tactic for solving the quadratic ideal-coupled problems.

5. NUMERICAL EXAMPLES

In this study, the finite element method has been employed for the key part of the analysis procedure. To reach this goal, a computer program was developed based on the true coupled, decoupled, ideal-coupled and quadratic ideal-coupled methods. The solid finite elements are applied for modeling the dam, and the fluid domain is discretized by the fluid finite elements. In what follows, to compare these methods, they are employed for finding the natural frequencies of three famous Iranian arch dams. Firstly, the finite element models of these dams and their basic parameters are introduced.

The water domain is considered as a region which extends to a specific length. It should be added that H is the dam height or the maximum water depth in the reservoir which is measured in upstream direction at the dam mid-crest point, and L denotes the water region length. Herein, it is assumed that $L = 0.2H$. It should be mentioned that the impounded water is considered as inviscid and compressible fluid with a unit weight equal to $9.81\,\mathrm{kN/m^3}$, and pressure wave velocity $C = 1440\,\mathrm{m/s}$.

5.1 Karun-3 arch dam

Firstly, the natural frequencies of the Karun-3 arch dam is obtained. For this purpose, 20-node isoparametric fluid and solid finite elements are applied. Number of the fluid and solid degrees of freedom are equal to 1158 and 576, respectively. The finite element mesh of the dam-reservoir system included 420 fluid and solid elements. For this dam, it is assumed that the Elasticity modulus $(E_d) = 23.6\,\mathrm{GPa}$; Poisson's ratio $= 0.2$ and Unit weight $= 24.5\,\mathrm{kN/m^3}$. Figure 1 shows the finite element model of this dam.

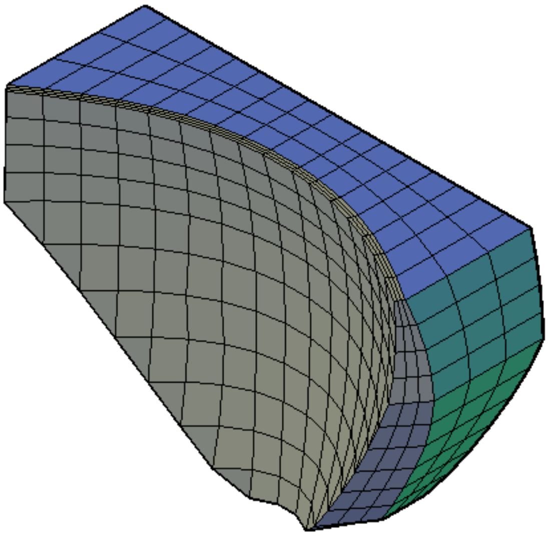

Figure 1 : The finite element model of the Karun-3 dam

Now, the first five natural frequencies of this dam are presented in Tables 1 and 2.

Table 1 : The first five natural frequencies of the Karun-3 dam-reservoir system

Mode Number	**Natural frequencies f_i (Hz)**			
	Decoupled	**Ideal-coupled**	**Quadratic Ideal-coupled**	**True coupled**
	Dam	**First ideal case (incompressible fluid assumption)**	**First ideal case**	
1	2.14	1.53	1.43	1.40
2	2.53	2.48	1.53	1.53
3	3.51	2.78	2.36	2.25
4	4.06	3.39	2.74	2.71
5	4.75	3.97	3.21	2.98

Table 2 : The first five natural frequencies of the Karun-3 dam-reservoir system

Mode Number	**Natural frequencies f_i (Hz)**			
	Decoupled	**Ideal-coupled**	**Quadratic Ideal-coupled**	**True coupled**
	Reservoir	**Second ideal case (Massless solid assumption)**	**Second ideal case**	
1	2.58	1.52	1.42	140
2	3.78	2.02	1.68	1.53
3	5.23	3.05	2.56	2.25
4	6.30	3.72	3.17	2.71
5	7.51	3.98	3.50	2.98

It is clear that the quadratic ideal-coupled results are more accurate, in comparison to those of the decoupled and ideal-coupled ones.

5.2 Karaj arch dam

The second well-known Iranian arch dam whose natural frequencies are obtained is the Karaj dam. Similarly, 20-node isoparametric fluid and solid finite elements are applied in this procedure. To produce the mesh of this dam, 180 fluid and solid elements were used. Number of the fluid and solid degrees of freedom are equal to 1860 and 700, respectively. For this dam, it is assumed that the Elasticity modulus $\left(E_d\right) = 25.5\,\text{GPa}$; Poisson's ratio $= 0.17$ and Unit weight $= 24.03\,\text{kN}/\text{m}^3$. In Figure 2, the finite element model of this dam is illustrated.

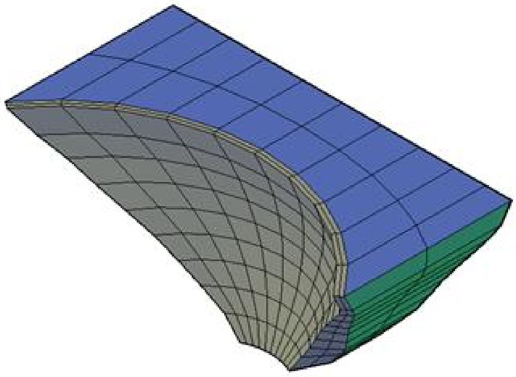

Figure 2 : The finite element model of the Karaj dam

Now, the first five natural frequencies of this dam are presented in Tables 3 and 4.

Table 3 : The first five natural frequencies of the Karaj dam-reservoir system

Mode Number	Natural frequencies f_i (Hz)			
	Decoupled	Ideal-coupled	Quadratic Ideal-coupled	True coupled
	Dam	First ideal case (incompressible fluid assumption)	First ideal case	
1	1.61	1.23	1.23	1.22
2	1.92	1.46	1.44	1.43
3	2.82	2.10	2.01	1.93
4	3.08	2.51	2.50	2.39
5	3.68	2.66	2.53	2.50

Table 4 : The first five natural frequencies of Karaj dam-reservoir system

Mode Number	Natural frequencies f_i (Hz)			
	Decoupled	Ideal-coupled	Quadratic Ideal-coupled	True coupled
	Reservoir	Second ideal case (Massless solid assumption)	Second ideal case	
1	2.68	1.83	1.41	1.22
2	4.51	1.86	1.61	1.43
3	5.62	2.30	2.16	1.93
4	7.06	2.59	2.96	2.39
5	8.65	3.39	3.20	2.50

It is clear that the quadratic ideal-coupled results are more accurate, in comparison to those of the decoupled and ideal-coupled ones.

5.3 Shahid Rajaee Arch Dam

Finally, the natural frequencies of an ideal model of the Shahid Rajaee arch dam are computed. In an analogous manner, 20-node isoparametric fluid and solid finite elements are applied. To generate the mesh of this dam, 120 fluid and solid elements were used. Number of the fluid and solid degrees of freedom are equal to 1860 and 700, respectively. For this dam, it is assumed that the Elasticity modulus $\left(E_d\right) = 30\,\text{GPa}$; Poisson's ratio $= 0.18$ and Unit weight $= 24\,\text{kN}/\text{m}^3$. In Figure 3, the finite element model of this dam is depicted.

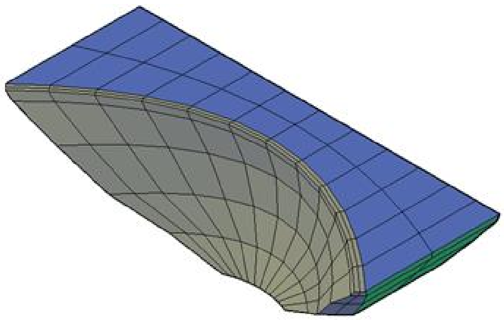

Fig. 3 : The finite element model of the Shahid Rajaee dam

Now, the first stage, the first five natural frequencies of this dam are presented in Tables 5 and 6.

Table 5 : The first five natural frequencies of the Shahid Rajaee dam-reservoir system

Mode Number	Natural frequencies f_i (Hz)			
	Decoupled	Ideal-coupled	Quadratic Ideal-coupled	True coupled
	Dam	First ideal case (incompressible fluid assumption)	First ideal case	
1	2.48	1.96	1.94	1.94
2	3.01	2.22	2.13	2.09
3	3.73	2.98	2.91	2.85
4	4.89	4.11	3.87	3.54
5	5.42	4.13	4.06	4.03

Table 6 : The first five natural frequencies of the Shahid Rajaee dam-reservoir system

Mode Number	Natural frequencies f_i (Hz)			
	Decoupled	Ideal-coupled	Quadratic Ideal-coupled	True coupled
	Reservoir	Second ideal case (Massless solid assumption)	Second ideal case	
1	2.28	2.48	2.22	1.94
2	3.54	2.87	2.26	2.09
3	4.21	4.31	3.50	2.85
4	4.86	5.18	4.51	3.54
5	4.86	5.71	5.06	4.03

It is clear that the quadratic ideal-coupled results are more accurate, in comparison to those of the decoupled and ideal-coupled ones.

6. CONCLUSIONS

In this paper, the quadratic-ideal coupled method was used for finding the eigen-pairs of three famous Iranian arch dams, including Karun-3, Karaj and Shahid Rajaee dams. For comparison, the results of the decoupled, ideal-coupled and the actual true coupled methods were presented. Findings show that the quadratic-ideal coupled natural frequencies are more close to the true ones, in comparison to the decoupled and ideal-coupled natural frequencies. As a consequence, using the mode shapes of the quadratic ideal-coupled approach in dynamic analysis may lead to more accurate results.

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