SHAPE DESIGN OF ARCH DAMS USING PARAMETERIZED CURVES AND ITS APPLICATION

C.H. HE, C. ZHANG AND J. ZHANG
Chengdu Engineering Corporation Limited, Chengdu, Sichuan Province, China

ABSTRACT

Conventionally the shapes of arch dams are characterized by a series of conic or logarithmic spiral curves. However, too many kinds of curve expressions bring problems to global optimization of dam shapes and integral design. In this paper a universal way describing and designing the arch dam shapes by using parameterized curves - Bezier curves is introduced. In summary, all the horizontal characteristic curves of the dam body are determined by fixed number of controlling points. It is proved that just one controlling point for each curve is enough for satisfying the engineering requirements, and shows a better fitting effect over the unified quadratic curves. In the designing or optimizing procedure, the dam shape adjustment, including the switching among conventional dam shapes or searching the global optimum shape, can be easily achieved by modifying the coordinates of the controlling points. An example of arch dam shape switching from parabolic type to circular type is presented. Simultaneously, the method for searching the optimal locations of the controlling points is developed. Combined with the FEM integral rapid design techniques and tools, the approach can offer dam engineers a broader perspective of the arch dam shape selection and a more efficient path finding the most suitable dam shape. An example of arch dam shape searching along the pre-determined route based on FEM integral design is given subsequently. The conclusions are finally drawn that the Bezier curve design technique for arch dam shape design shows a better fitting effect than other curves, offers a more convenient way for arch dam shape changing and searching in FEM-based integral design.

1. INTRODUCTIONS

1.1 Arch dam shape design

For double-curvature arch dams, the shapes of characteristic curves are generally described in horizontal and vertical sections respectively. Since in the vertical section the curves are conventionally denoted by cubic polynomials, the arch dam shapes are basically featured by the horizontal curves. In early designs for arch dam shape the circles are adopted, and noncircular curves including parabola, ellipse, hyperbola, three-center circle are used since the 1950s as the dam construction technique develops (Sun, 2008). However handling different curve equations brings many problems to engineers, researchers thus introduce a series of curve equation in uniform format, such as unified quadratic curve (Li, 1988). Unfortunately in the arch dam designing practice, engineers still tend to focus on a single curve type, mostly due to the restriction of computation tools and means.

1.2 Integration of design, analysis and evaluation

With the fast development of computer software and hardware artificial, it has been a trend to solve engineering design problems by way of massive numerical computation as well as rapid real-time feedback. Hence, fast modeling is required. This technology demands us to build a connection between the mathematic equations and the 3D arch dam model, and meanwhile, this approach should compatible with different arch dam types, namely, an universal expression based on computer graphic theory for different curves need to be developed.

1.3 Approach of this paper

In order to unify all the curve types so as to realize fast modeling-analysis-integration, this paper introduces an approach: (1) Firstly the Bezier curve is applied instead of traditional mathematic curves. For most cases the Bezier curve doesn't strictly equals all types of curves, however errors are considered acceptable in engineering scope when small enough and meanwhile the order (namely the number of controlling points) of the Bezier curve is determined. (2) Secondly the

Bezier curves form Bezier surfaces (Mortensen, 1985), and eventually form the solid of the arch dam. (3) Finally in the arch dam shape design the dam type switching and the optimum shape searching can be achieved by just change the coordinates of the controlling points, without redefining the curve equations or changing the program structure.

2. METHOD

2.1 Bezier Curves

Published in 1962 by French engineer Pierre Bezier, the Bezier curve was firstly used in the automobile design (Bezier, 1990). And with the rapid development of computer technology afterwards, it was widely applied in computer graphics aiding designers to draw smooth curves. The general expression of Bezier curve is written as

$$B(t)=\sum_{i=0}^{n}\binom{n}{i}P_i(1-t)^{n-i}t^i, t\in[0,1] \quad ...(1)$$

where $B(t)$ is the Bezier curve with the parameter t, P_i is the ith controlling point and generally P_0 is the start point and P_n is the end point, n is the order of the curve.

From the general expression it is easy to know that the 1-order, 2-order and 3-order Bezier curves are:

1-order: $$B(t)=(1-t)P_0+tP_1, t\in[0,1] \quad ...(2)$$

2-order: $$B(t)=(1-t)^2P_0+2t(1-t)P_1+t^2P_2, t\in[0,1] \quad ...(3)$$

3-order: $$B(t)=(1-t)^3P_0+3t(1-t)^2P_1+3t^2(1-t)P_2+t^3P_3, t\in[0,1] \quad ...(4)$$

2.2 Fitting conventional curves using Bezier curves and unified quadratic curves

From the *Handbook for Hydraulic Structure Designing* (Zhou and Dang, 2011) in China, the following curve forms are mentioned that are adoptable for horizontal arch rings in dam shape designing: circle, three center circle, parabola, logarithmic spiral, ellipse and unified quadratic curve. classified by the expression forms of the above curves, three categories are presented: (1) The three center circle is expressed by piecewise function (Equation 5); (2) The circle, the parabola and the ellipse actually belong to the unified quadratic curves, subjecting to the general form of Equation 6; (3) The logarithmic spiral can be list as one kind of the particular curves, see Equation 7.

$$\begin{cases} x^2+(y-y_1)^2=R_1^2, x\in[x_0,x_1] \\ x^2+(y-y_2)^2=R_2^2, x\in[x_1,x_2] \\ x^2+(y-y_3)^2=R_3^2, x\in[x_2,x_3] \end{cases} \quad ...(5)$$

whereR_1, R_2, R_3 are radiuses of three circles respectively.

$$x^2+ay^2+by+c=0, x\in[x_0,x_1] \quad ...(6)$$

where when a = 0 Equation 6 degenerates to parabolic equation, when a = 1 to circle equation, when a > 0 and $a \neq 1$ to ellipse equation, and when b < 0 to hyperbolic equation.

$$\rho=\rho_0 e^{a\theta}, \theta\in[\theta_0,\theta_1] \quad ...(7)$$

where ρ_0 is the polar radius at the arch crown and a is a coefficient.

Now take the Dagangshan arch dam on the Dadu River in Western China as an example to validate the precision of the fitting by Bezier curves and unified quadratic curves. As is shown in Figure 1, the 210 m-high Dagangshan arch dam is a double-curvature arch dam with parabolic horizontal arch rings, among which the right half of the central line of the top arch starts from Point A (0m, 295m) to Point B (-281.25m, 147.15m).

We replace the parabola between Point A and Point B with circle, three center circle, ellipse, logarithmic spiral and then obtain five different basic curves in total, to compare the fitting effects of the unified quadratic curve and Bezier curve.

Given that the starting point and ending point are fixed, the circle, the parabola and the logarithmic spiral are completely determined, whereas the three center circle and the ellipse still vary. The equations of three center circle and ellipse are given as Equation 8 and Equation 9 with additional conditions. In both equations the variable θ is depicted in Figure 1.

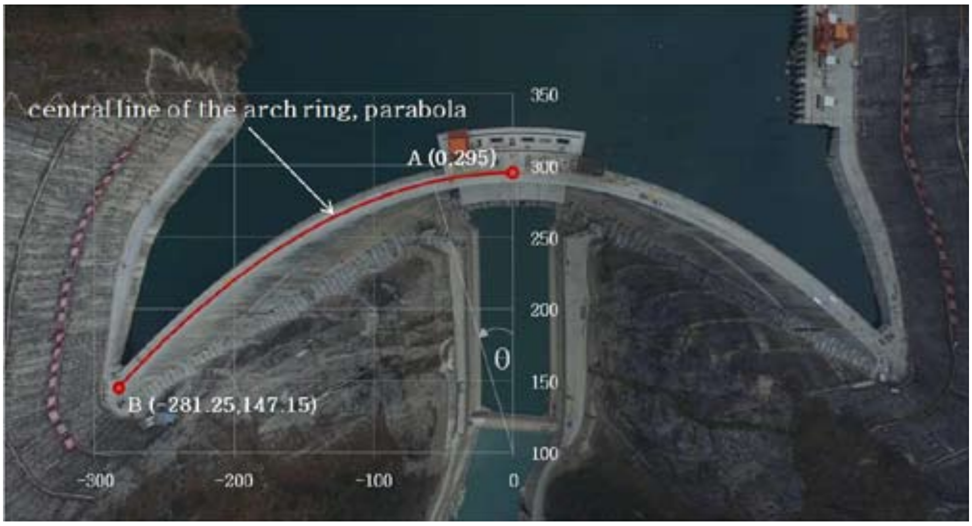

Figure 1 : Top view of the Dagangshan arch dam and its arch ring

$$\begin{cases} x = -198.79\sin\theta \\ y = 96.21 + 198.79\cos\theta \end{cases} \quad \theta \in [0,20]$$
$$\begin{cases} x = 131.80 - 584.13\sin\theta \\ y = -265.90 + 584.13\cos\theta \end{cases} \quad \theta \in (20,45] \qquad \text{...(8)}$$

$$\begin{cases} x = -437.16\sin\theta \\ y = -336.96 + 631.96\cos\theta \end{cases} \quad \theta \in [0,40] \qquad \text{...(9)}$$

Then the error function σ is built as

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}\left[f(x_i) - g(x_i)\right]^2}{N}}, x_i \in [-281.25, 0] \qquad \text{...(10)}$$

where $f(x)$ and $g(x)$ are respectively the fitting curve and original curve, N is the number of data points on the curve.

Since the starting and ending points are given, there remains only one free coefficient in Equation 6, and thus we take the coefficient a varying between interval [-2, 4] and calculate the errors between the unified quadratic curve and other curves based on Equation 10. As mentioned above, the parabola, the ellipse and circle can be strictly equaled, therefore while a takes certain value the errors for fitting the three curves are expected as 0. Through simple calculation it is figured out that a equals 0, 0.47 and 1, respectively. However while fitting the three center circle and logarithmic spiral the error cannot be totally eliminated as the minimum value are 2.4 and 0.78. The errors for fitting the five curves are plotted in Figure 2.

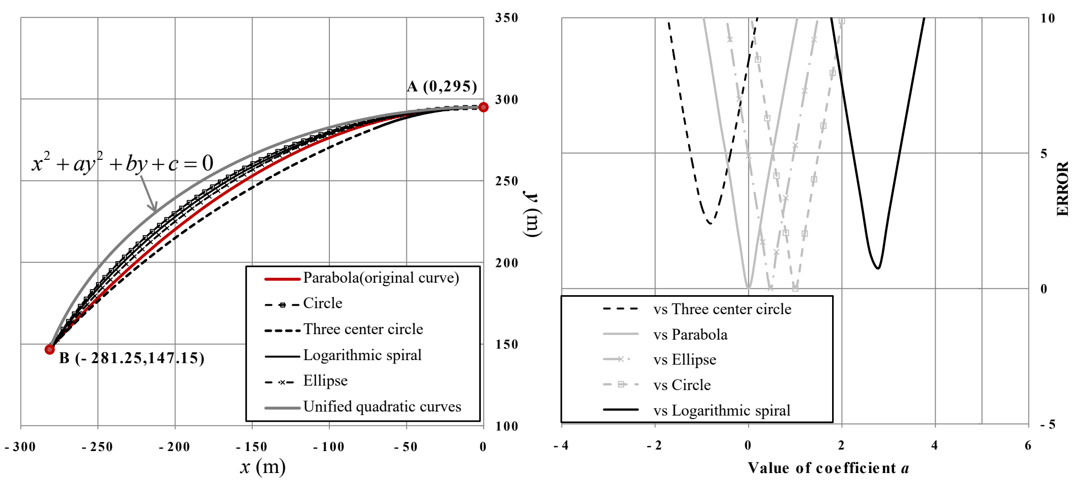

Figure 2 : Unified quadratic curve fitting and its errors

When using the Bezier curve to fit the other curves we just need to move the controlling point P1 as shown in Figure 3.

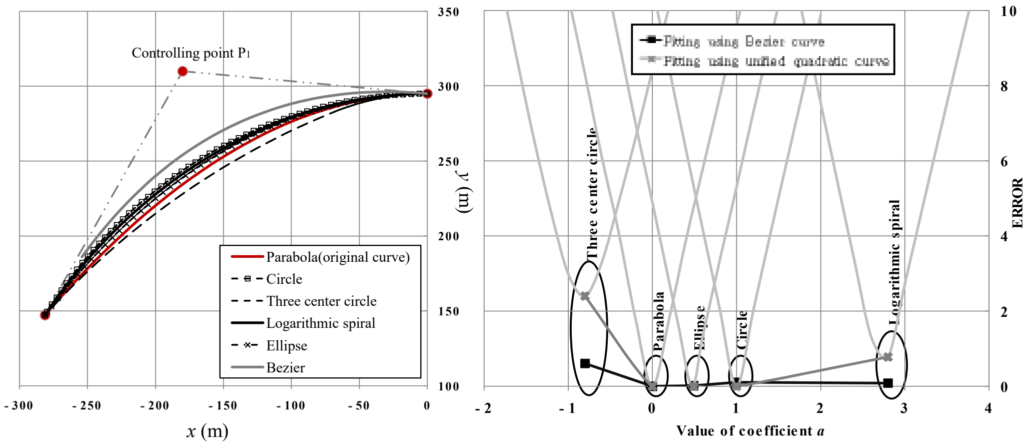

Figure 3 : Bezier curve fitting and its errors compared with unified quadratic curve

The coordinate plane is discretized into $\Delta x \times \Delta y = 0.1\text{m} \times 0.1\text{m}$ grids and the fitting results are given in Table 1. The right half of Figure 3 compares the fitting error of the unified quadratic curve and Bezier curve. Although in all the other cases except fitting parabola, Bezier curves cannot strictly equal the object curves, the overall errors are obviously smaller than those of unified quadratic curves. Moreover, to improve the fitting effect of Bezier curves, there are two approaches available: (1) Refine the coordinate plane grids of the interested area; (2) Increase the order of Bezier curves, namely, increase the number of controlling points. Both approaches are flexible and effective to be implemented in an integral design of the arch dam shape.

Table 1 : Fitting error using Bezier curves

Object curves	Coordinate of P1	Fitting error σ
Parabola*	(-140.6, 295.0)	0.000
Circle	(-178.4, 290.6)	0.109
Three center circle	(-127.1, 290.1)	0.608
Logarithmic spiral	(-170.0, 291.0)	0.084
Ellipse	(-158.0, 293.2)	0.017

*When x-coordinate of P1 equals 0.5 times x-coordinate of B, the 2-order Bezier curve strictly equals parabola.

3. APPLICATION IN INTEGRAL DESIGN

3.1 FEM-based integral design

As is shown in Figure 4, for the integral design of arch dams there are four main steps in one loop. In our previous work, the process from geometric model to FEM analysis has been completed. Thus this paper focuses on the first step, namely, fast geometric modeling from parameters to arch dam model. With the method introduced in this paper, all the arch dam shape changing could be achieved by just moving the positions of the controlling points. Moreover, from the results of error analysis, only one controlling point is needed for each curve.

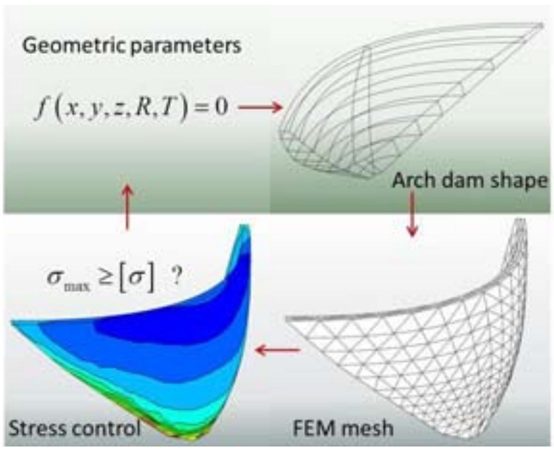

Figure 4 : Schematic map of FEM-based integral design, where $f(x,y,z,R,T) = 0$ is the shape function of the arch dam and x, y, z, R, T are x-, y-, z-coordinate, radius of curvature and thickness, and σ_{max}, $[\sigma]$ are maximum (tensile or compressive) stress and design stress.

3.2 Application 1: shape switching

It is common to comparatively analysis different horizontal curves in arch dam shape design when the boundary conditions (determined by canyon topography and rock quality) are given. With traditional method, designers have to change the equations, or even use another program. In this paper, we take the shape switching from parabola to circle as an example. As shown in Figure 5, the red points are endpoints located on the crown cantilever and the base surface, and the green points are controlling points. When these controlling points take the coordinate values shown in the second column in Table 2, the dam remains parabolic double-curvature arch dam, whereas the points take the values of the third column, the dam turns into circular double-curvature arch dam. The comparison between the parabolic and the circular arch dam is illustrated in Figure 6.

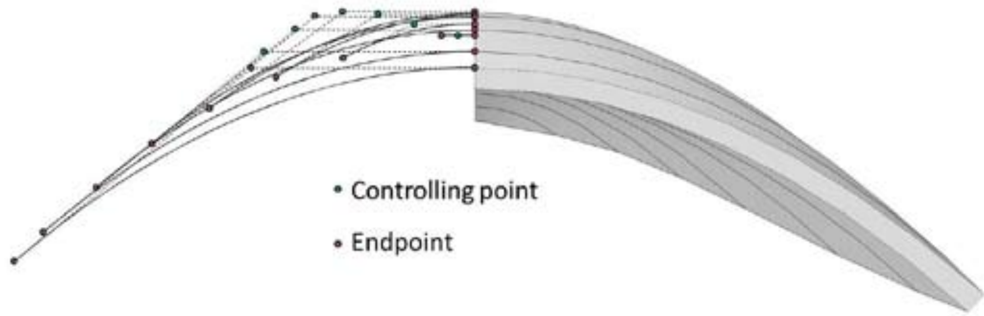

Figure 5 : Controlling points of the characteristic curves of the dam

For other curve types such as ellipse, multi-center circle, logarithmic spiral, the technical routes are the same with that of the circle applied hereby. It is worth noting that the controlling point coordinate values are calculated through fitting error minimization (least square method) between the original and Bezier curves.

Table 2 : Coordinates of controlling points (denoted as P1) for different arch dam shape

Characteristic elevation (m)	Coordinate of P1 (parabola) (m)	Coordinate of P1 (circle) (m)
2894	(-127.05, 300.00)	(-149.06, 298.00)
2870	(-119.15, 308.85)	(-140.18, 306.85)
2830	(-104.67, 320.57)	(-123.20, 318.57)
2790	(-89.34, 327.85)	(-102.82, 326.85)
2760	(-73.37, 329.95)	(-82.85, 328.96)
2730	(-55.05, 328.83)	(-62.05, 327.82)
2700	(-36.48, 324.17)	(-41.98, 323.16)
2677	(-9.72, 318.00)	(-11.25, 317.98)

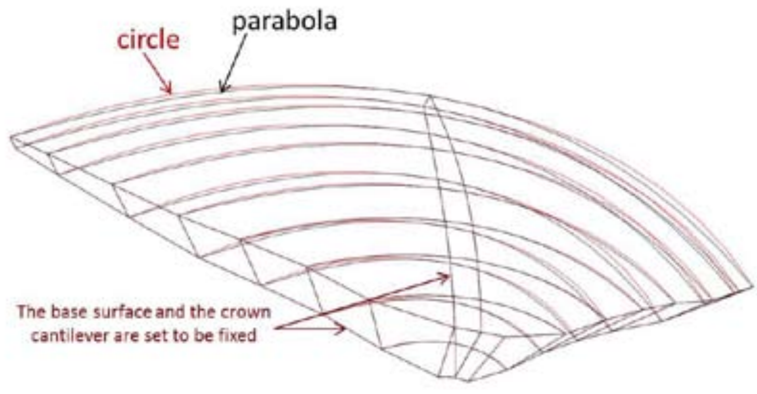

Figure 6 : Comparison between the circle- and parabola-shaped arch dam

3.3 Application 2: optimum shape searching

From the study in Chapter 3.2 it can be inferred that when the coordinate values of controlling points take certain specific numbers, the dam shape turns into double-curvature parabolic, circular, or elliptical, multi-centered arch dam shapes and so on. Besides the specific discrete coordinate sequences corresponding to traditional dam shapes, there exists many shapes which cannot be described by general mathematic equations actually. Therefore it is possible that the designers only figure out local optimum dam shape.

Searching the optimal shape using Bezier curve expression can effectively avoid the risk of locally optimal solution and surely obtain the global optimal solution theoretically. A schematic example is studied in this chapter. As given in Figure 7, a pre-supposed transverse search route is given, stretching from the crown cantilever to the right base surface. Note that Figure 7 only illustrates the top arch ring. In fact, all the other arch rings at different elevation subject to a group of parallel search routes.

The search route between SP1 and SPn is equally divided into 13 segments, namely, 12 controlling points uniformly distributed on the route, numbered as ①~⑿, corresponding to 12 different arch dam shapes. With totally same loads, material parameters and foundation model applied, the stress results are calculated using FEM, as shown in Figure 8. Furthermore, the values are listed in Table 3. For each dam shape, the minimum and maximum principal stresses are given. Considering that the mesh sensitivity may impact the results as the dam shape changes, the compressive and tensile ratios are also listed, representing the global stress level of the dam.

From the results it can be seen that when t-value equals 6/13 the minimum principal stress and the compressive as well as tensile ratios are all the lowest, and when equals 5/13 the maximum principal stress gets the lowest. Exclude the influence of the mesh sensitivity, it can be concluded that when the controlling point is located at the middle of SP1 and SPn, the stress state of the dam body is the most advantaged. That is to say, the parabolic double-curvature arch dam is the optimum dam shape on search route SP1-SPn.

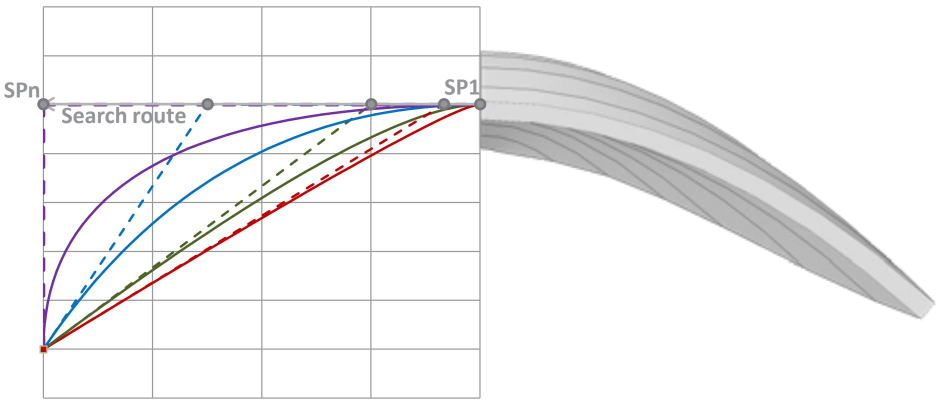

Figure 7 : Arch dam shape searching

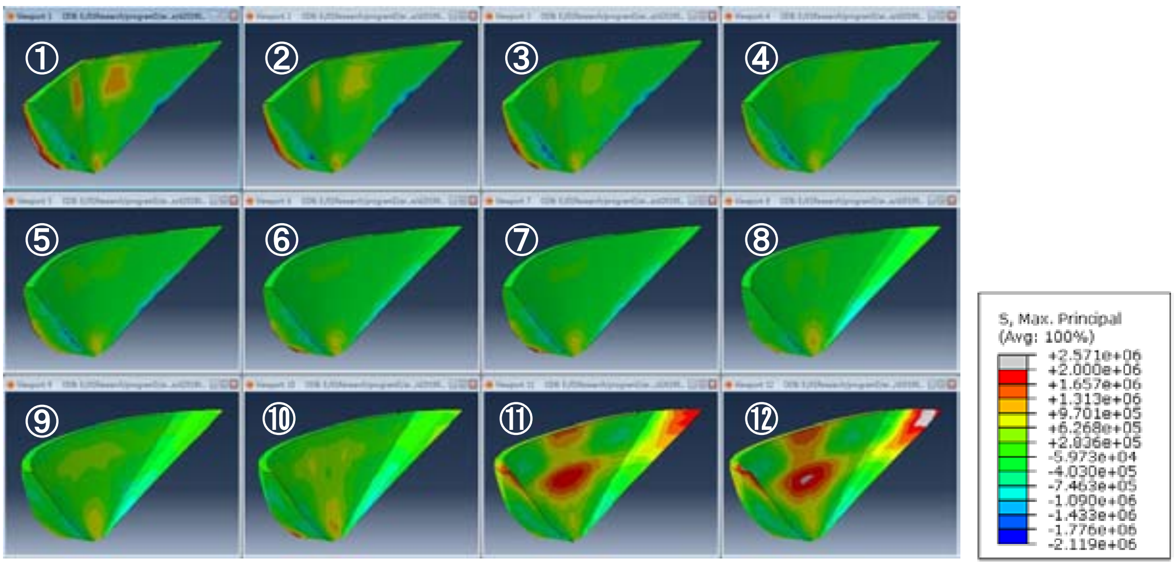

Figure 8 : Stress results of the 12 arch dam shapes (Maximum principal stress, unit: Pa)

Table 3 : Stress results of the arch dam shape searching

Shape No.	t-value of the controlling point	Min. principal stress (MPa)	Max. principal stress (MPa)	Compressive ratio	Tensile ratio
1	1/13	-11.98	1.88	1.24%	35.78%
2	2/13	-11.24	1.72	1.37%	34.25%
3	3/13	-10.96	2.13	0.95%	31.21%
4	4/13	-9.86	1.59	0.72%	27.70%
5	5/13	-9.89	1.69	0.71%	21.86%
6	6/13	-9.43	1.72	0.45%	17.71%
7	7/13	-10.33	2.20	0.86%	19.09%
8	8/13	-10.36	2.17	0.87%	20.27%
9	9/13	-10.46	2.32	2.16%	28.35%
10	10/13	-11.18	2.52	3.46%	35.36%
11	11/13	-11.12	1.87	3.10%	41.22%
12	12/13	-11.79	2.35	4.08%	40.98%

Note: 1.t-value of the controlling point refers the ratio between the controlling-point-SP1 distance and SPn-SP1 distance.

2. The compressive and tensile ratio respectively defined as percentages of nodes where minimum stress lower than 8.0 MPa and maximum stress higher than 2.0 MPa in the whole dam body.

4. DISCUSSIONS AND CONCLUSIONS

4.1 Discussions

In this paper when fitting the object curves, the determination of the controlling point location is completed with computer program by traversing all the grid nodes. And the process is repeated for all the characteristic curves, which will cost fairly long time. Actually for general mathematic curves, explicit expressions of the controlling point coordinates seem can be derived, or the traversing area can be dramatically narrowed through some regulation research. In addition, the searching for the controlling point in a designated area (if given) should be optimized instead of simply grid-by-grid. All the work mentioned above is plan to be carried out in the next stage follows.

4.2 Conclusions

This paper introduced Bezier curves to the arch dam shape design, offering an effective technical route for the successful implementation. Through some fundamental research and two application case study, we can draw the following conclusions:

Compared to the unified quadratic curves, Bezier curves can accommodate more curve types. The fitting error can basically keep stable and stay underneath a relatively low level just in the second order. To obtain higher fitting precision, we just need to add more controlling points.

In the FEM-based integral design, Bezier curve takes an obvious advantage over all the other curves in the geometric modeling of the arch dam. All the shape adjustment could be completed by change the coordinate values, as a result, the program structure can remain unchanged and the integral design gets faster and more convenient.

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