Evaluation of Water Table Profile Through Earth-fill Dams by GaussQuadrature Integral Method

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Abstract

The seepage problem in porous media and finding hydraulic properties of flow, is one of the most attractive subjects in the scientific field of porous media. Among these, evaluation of free surface profile through earth-fill dams with known water depth at both upstream and downstream reservoirs, is a major problem being considered in different ways by many researchers. Up to now, several numerical methods such as finite difference method, finite element method, finite volume method and some innovative methods (e.g., simulation by equivalent pipe network which is performed by Abareshi et al., 2017) utilizing commercial software or special purpose programs, have been applied to solve the aforementioned problem. However, it is interesting that there is an exact mathematical solution to this problem which is presented by Polubarinova-Kochina (1962). The main objective of this article is to consider this exact solution and re-evaluating some of relevant calculations by Gauss quadrature integral method. In fact, Polubarinova-Kochina solution process includes some mathematical concepts and methods such as hodograph method, Schwarz-Christoffel transformation, complex analytic functions, hypergeometricGauss function and etc., which are frequently expressed by some integrals. In this paper, we would try to calculate these integrals by Gauss quadrature method. Moreover, the results of this research are compared and verified with the results of studies done by previous researchers.

Keywords: porous media, free surface profile, Polubarinova-Kochina exact solution, Gauss quadrature integral method.

1. INTRODUCTION

In different engineering projects such as environmental, hydraulics and civil engineering, in which flow through porous media is involved, seepage flow analysis plays a major role in hydraulic structures like dams and embankments. So problems related to slope stability and structure failure caused by seepage, are affected critically by seepage evaluation, which is achieved by solving governing equation.

There have been many studies conducted to solve this problem by different numerical methods. For example,Hosseini and Sonei (2003) have solved governing equation of flow through rockfills by finite element method in a fixed grid [1]. Other researchers such as Tayfur et al. (2005), Soleimani and Akhtarpour (2011), Wang et al. (2015) have applied finite element method to solve such problems [2,3,4]. Finite difference method and finite volume method have also been utilized successfully by Jie et al. (2004), Kermani and Barani (2012), Darbandi et al. (2007) and Bresciani et al. (2012), etc[5,6,7,8].

In recent years, many other innovative methods have been applied to solve seepage problems. For instance, Abareshi et al. (2017) have implemented an equivalent pipe network model (EPNM) for analysis of nonlinear two-dimensional steady flow with free surface in a homogeneous isotropic coarse porous media [9]. Hayek (2019) has presented a new mathematical method as an approximate solution to the problem of flow caused by sudden changes of boundary heads in horizontal aquifers [10].Kazemzadeh-Parsi (2019) has considered the numerical solution offree surface flow problems in heterogeneous porous media [11].Kermani et al. (2019) have proposed mesh-free approach called smooth particle hydrodynamics (SPH) to solve governing equation of seepage flow in complex geometry problems [12].

Besides old and recent studies in this field, Polubarinova-Kochina (1962) could find some formulas to provide the exact solution to the problem of steady-state flow with free surfacethrough a rectangular dam [13]. The work done by her is very worthy due to its usefulness for verificationof other approximate methods. Nevertheless,Polubarinova-Kochina's formulas were rarely used as a referenceto evaluate the results of numerical methods. Maybe, the reason is that these formulas are not very easy to use. Hornung and Krueger (1985) used Polubarinova-Kochina's formulas and computed the complex integrals by Romberg's rule

[14].Theypresented exact numerical values in format of tables and graphs, which had not been valid before, for some sample cases.These results involve height of seepage face and the shape of the free surface of the flow passing through a dam.

In this article, in a research similar to the study of Hornung and Krueger (1985), we consider Polubarinova-Kochina's formulas for exact solution of free surface flow through porous media and useGauss quadrature integral methodto compute the related integrals. This method can somehow eradicate the difficulty of calculating these integrals. Then, we compare our results with those of Hornung and Krueger (1985).

2. POLUBARINOVA-KOSHINA'S FORMULAS

The general seepage problem considered by Polubarinova-Kochina (1962) has been described in Figure 1.According to Figure 1, between two lakes of heights H_1 and H, There is a dam with homogeneous porous media of width L Above an impervious layer. The upper boundary, Γ, of the domain, Ω, is the free (pheratic) surface, location of which should be determined.

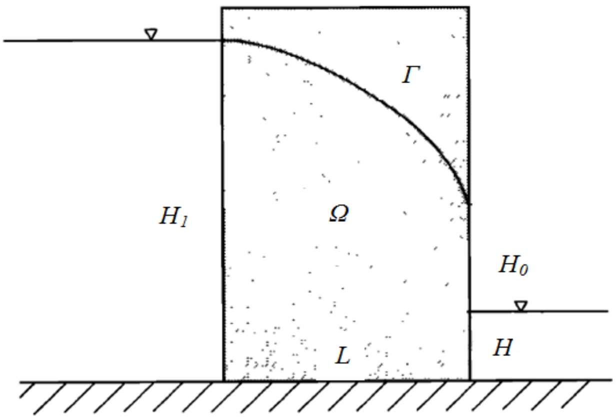

Figure 1. Cross section of the dam considered by Polubarinova-Kochina (1962) [14]

Polubarinova-Kochina (1962) applied the theory of linear differential equations to this problem and used the hodograph and complex potential planes and the solutions of the hypergeometric equationsto derive someformulas which contribute to calculating the height of the seepage face of the dam as followings [13]:

$$L = C\int_0^{\frac{\pi}{2}} \frac{K(\alpha + (\beta-\alpha)\,\mathrm{Sin}^2(\varphi))}{\left(1-\alpha-(\beta-\alpha)\,\mathrm{Sin}^2(\varphi)\right)^{\frac{1}{2}}}\,d\varphi \tag{1}$$

$$H = C\sqrt{\alpha}\int_0^{\frac{\pi}{2}} \frac{K(\alpha\,\mathrm{Sin}^2(\varphi))\,\mathrm{Sin}(\varphi)}{\left((1-\alpha\,\mathrm{Sin}^2(\varphi))(\beta-\alpha\,\mathrm{Sin}^2(\varphi))\right)^{\frac{1}{2}}}\,d\varphi \tag{2}$$

$$H_0 = C\int_0^{\frac{\pi}{2}} \frac{K(\alpha\,\mathrm{Cos}^2(\varphi))\,\mathrm{Sin}(\varphi)\,\mathrm{Cos}(\varphi)}{\left((1-\alpha_1\,\mathrm{Sin}^2(\varphi))(1-\beta_1\,\mathrm{Sin}^2(\varphi))\right)^{\frac{1}{2}}}\,d\varphi \tag{3}$$

$$H_1 = H + H_0 + C\int_0^{\frac{\pi}{2}} \frac{K(\mathrm{Cos}^2(\varphi))\,\mathrm{Sin}(\varphi)}{\left((1-\alpha\,\mathrm{Sin}^2(\varphi))(1-\beta\,\mathrm{Sin}^2(\varphi))\right)^{\frac{1}{2}}}\,d\varphi \tag{4}$$

Where, α, β and C are constants which should be determined. α and β should satisfy the conditions $0 \le \alpha < \beta < 1$ and $\alpha_1 = 1-\alpha$, $\beta_1 = 1-\beta$ and $K(k)$ is the complete elliptic integral of the first kind and defined by the following equation [13]:

$$K(k) = \int_0^{\frac{\pi}{2}} \frac{d\varphi}{\left(1-k\,\mathrm{Sin}^2(\varphi)\right)^{\frac{1}{2}}} \tag{5}$$

It should be mentioned that by choosing an appropriate value for C, H_1 will equal 1 and other dimensions (H_0, H and L) will be scaled the same as H_1. For doing the analysis, α and β should be determined for a special case by solving equations (1) and (2) simultaneously. After specifying these parameters, the seepage face height, H_0, can be calculated by equation (3)[14].

In the next section, we define Gauss quadrature integral method which is used to deal with equations (1)-(5) in this article.

3. GAUSS QUADRATURE INTEGRAL METHOD

Gauss quadrature method is an approximate way to compute complex integrals which cannot be computed easily by exact methods. This method is defined below [15]:

$$\int_{-1}^{1} f(\xi)\, d\xi = \sum_{i=1}^{ng} W_i f(\xi_i) \tag{6}$$

Where, f is a function, integral of which is to be calculated, ng is the number of key points that are located in the domain of integration, ξ_i is the i th key point and W_i is the weight of each key point ξ_i.

It should be noted that Gauss method with ng key points,can compute the exact integral of polynomials with maximum exponent of $2ng-1$. For the functions which are not in the form of a polynomial, the more key points are selected, the more exact the result will be.

Here,for $ng = 4$, we try to compute ξ_i s and corresponding W_i s.

As stated before, for $ng = 4$, the integral of a polynomial with exponent of $2ng-1=7$ can be calculated exactly. So we have:

$$f(\xi) = a\xi^7 + b\xi^6 + c\xi^5 + d\xi^4 + e\xi^3 + f\xi^2 + g\xi + h \tag{7}$$

Where, $a, b, c, d, e, f, g,$ and h are arbitrary coefficients of a polynomial with exponent of 7.

The exact integral of the function in (7) can be computed easily as the following:

$$\int_{-1}^{1} f(\xi)\, d\xi = 0a + \frac{2}{7}b + 0c + \frac{2}{5}d + 0e + \frac{2}{3}f + 0g + 2 \tag{8}$$

On the other hand, Gauss integral of this function for $ng = 4$ is obtained according to (6) as:

$$\sum_{i=1}^{ng=4} W_i f(\xi_i) = W_1 f(\xi_1) + W_2 f(\xi_2) + W_3 f(\xi_3) + W_4 f(\xi_4) \tag{9}$$

Forequation(8) being equal to equation (9), 8 equations which are mentioned below, should be satisfied:

$$W_1\xi_1^7 + W_2\xi_2^7 + W_3\xi_3^7 + W_4\xi_4^7 = 0 \tag{10}$$
$$W_1\xi_1^6 + W_2\xi_2^6 + W_3\xi_3^6 + W_4\xi_4^6 = \frac{2}{7} \tag{11}$$
$$W_1\xi_1^5 + W_2\xi_2^5 + W_3\xi_3^5 + W_4\xi_4^5 = 0 \tag{12}$$
$$W_1\xi_1^4 + W_2\xi_2^4 + W_3\xi_3^4 + W_4\xi_4^4 = \frac{2}{5} \tag{13}$$
$$W_1\xi_1^3 + W_2\xi_2^3 + W_3\xi_3^3 + W_4\xi_4^3 = 0 \tag{14}$$
$$W_1\xi_1^2 + W_2\xi_2^2 + W_3\xi_3^2 + W_4\xi_4^2 = \frac{2}{3} \tag{15}$$
$$W_1\xi_1 + W_2\xi_2 + W_3\xi_3 + W_4\xi_4 = 0 \tag{16}$$
$$W_1 + W_2 + W_3 + W_4 = 2 \tag{17}$$

The unknowns existing in the system of equations (10)-(17) are $\xi_1, \xi_2, \xi_3, \xi_4, W_1, W_2, W_3$ and W_4, which can be obtained by solving these equations simultaneously. Finally the Gauss parameters are obtained as: $\xi_1 = 0.861136$, $\xi_2 = -0.861136$, $\xi_3 = 0.339981$, $\xi_4 = -0.339981$, $W_1 = 0.347855$, $W_2 = 0.347855$, $W_3 = 0.652145$ and $W_4 = 0.652145$.

Both W_i and ξ_i have been determined specifically for each value of ng in a table as the one presented below:

Table 1- key points and corresponding weights in Gauss quadrature method [15]

n	ξ_i	W_i
1	0. (15 zeros)	2. (15 zeros)
2	±0.57735 02691 89626	1.00000 00000 00000
3	±0.77459 66692 41483	0.55555 55555 55556
	0.00000 00000 00000	0.88888 88888 88889
4	±0.86113 63115 94053	0.34785 48451 37454
	±0.33998 10435 84856	0.65214 51548 62546
5	±0.90617 98459 38664	0.23692 68850 56189
	±0.53846 93101 05683	0.47862 86704 99366
	0.00000 00000 00000	0.56888 88888 88889
6	±0.93246 95142 03152	0.17132 44923 79170
	±0.66120 93864 66265	0.36076 15730 48139
	±0.23861 91860 83197	0.46791 39345 72691

In the next section, we try to compute integrals of Polubarinova-Kochina's formulas by Gauss method.

4. SEEPAGE FACE HEIGHT EVALUATION

Here, the exact solution to the seepage problemdefined by equations (1)-(4) is calculated by Gauss method for various values of L and H. But first, as mentioned before, α, β and C should be determined foreach value of L and H. To this end, we have solved equations (1), (2) and (4) simultaneously. To use this method, we need the lower and upper limits of the integrals to be respectively -1 and 1. So we have to transform the variable φ to ξ by the following mapping:

$$\varphi = \frac{\pi}{4}(\xi + 1) \rightarrow d\varphi = \frac{\pi}{4} d\xi \tag{18}$$

After implementing this transformation, each integral with a general function of $g(\varphi)$ would be changed as the following term and then we can use Gaussmethod as it is mentioned bellow:

$$\int_0^{\frac{\pi}{2}} g(\varphi)\, d\varphi = \frac{\pi}{4} \int_{-1}^{1} f(\xi)\, d\xi = \frac{\pi}{4} \sum_{i=1}^{ng} W_i f(\xi_i) \tag{19}$$

For examplefor $L = 0.5$ and $H = 0.5$ and by choosing $ng = 3$ for Gauss integral method, we derive the relevant equations here:

$$C \int_0^{\frac{\pi}{2}} \frac{K(\alpha + (\beta - \alpha)\mathrm{Sin}^2(\varphi))}{\left(1 - \alpha - (\beta - \alpha)\mathrm{Sin}^2(\varphi)\right)^{\frac{1}{2}}} d\varphi = L \rightarrow \frac{\pi}{4} C\left(W_1 f_1(\varphi_1) + W_2 f_1(\varphi_2) + W_3 f_1(\varphi_3)\right) = L \tag{20}$$

$$C\sqrt{\alpha} \int_0^{\frac{\pi}{2}} \frac{K(\alpha\,\mathrm{Sin}^2(\varphi))\,\mathrm{Sin}(\varphi)}{\left((1 - \alpha\,\mathrm{Sin}^2(\varphi))(\beta - \alpha\,\mathrm{Sin}^2(\varphi))\right)^{\frac{1}{2}}} d\varphi = H \rightarrow \frac{\pi}{4} C\left(W_1 f_2(\varphi_1) + W_2 f_2(\varphi_2) + W_3 f_2(\varphi_3)\right) = H \tag{21}$$

$$H+H_0+C\int_0^{\frac{\pi}{2}}\frac{K(\mathrm{Cos}^2(\varphi))\mathrm{Sin}(\varphi)}{\left((1-\alpha\,\mathrm{Sin}^2(\varphi))(1-\beta\,\mathrm{Sin}^2(\varphi))\right)^{\frac{1}{2}}}d\varphi=H_1\rightarrow\frac{\pi}{4}C\left(W_1f_2(\varphi_1)+W_2f_2(\varphi_2)+W_3f_2(\varphi_3)\right)$$

$$+\frac{\pi}{4}C\left(W_1f_3(\varphi_1)+W_2f_3(\varphi_2)+W_3f_3(\varphi_3)\right)+\frac{\pi}{4}C\left(W_1f_4(\varphi_1)+W_2f_4(\varphi_2)+W_3f_4(\varphi_3)\right)=H_1 \qquad (22)$$

Where, H and H_0 is considered as equations (2) and (3),respectively, φ_i is evaluated by equation (18) for each value of ξ_i ($i=1,2,3$)and $f_1(\varphi_i)$, $f_2(\varphi_i)$, $f_3(\varphi_i)$ and $f_4(\varphi_i)$ have been defined as the following:

$$f_1(\varphi_i)=\frac{\hat{K}\left(\alpha+(\beta-\alpha)\mathrm{Sin}^2(\varphi_i)\right)}{\left(1-\alpha-(\beta-\alpha)\mathrm{Sin}^2(\varphi_i)\right)^{\frac{1}{2}}} \qquad (23)$$

$$f_2(\varphi)=\frac{\hat{K}\left(\alpha\,\mathrm{Sin}^2(\varphi_i)\right)\mathrm{Sin}(\varphi_i)}{\left((1-\alpha\,\mathrm{Sin}^2(\varphi_i))(1-\beta\,\mathrm{Sin}^2(\varphi_i))\right)^{\frac{1}{2}}} \qquad (24)$$

$$f_3(\varphi)=\frac{\hat{K}\left(\alpha\,\mathrm{Cos}^2(\varphi_i)\right)\mathrm{Sin}(\varphi_i)\mathrm{Cos}(\varphi_i)}{\left((1-\alpha_1\,\mathrm{Sin}^2(\varphi_i))(1-\beta_1\,\mathrm{Sin}^2(\varphi_i))\right)^{\frac{1}{2}}} \qquad (25)$$

$$f_4(\varphi)=\frac{\hat{K}\left(\mathrm{Cos}^2(\varphi_i)\right)\mathrm{Sin}(\varphi_i)}{\left((1-\alpha\,\mathrm{Sin}^2(\varphi_i))(1-\beta\,\mathrm{Sin}^2(\varphi_i))\right)^{\frac{1}{2}}} \qquad (26)$$

Where, $\hat{K}(k)$ is the Gauss integral of the function $K(k)$ in equation (5) and is expressed as below:

$$\hat{K}(k)=\frac{\pi}{4}\left(\frac{W_1}{\left(1-k\,\mathrm{Sin}^2(\varphi_1)\right)^{\frac{1}{2}}}+\frac{W_2}{\left(1-k\,\mathrm{Sin}^2(\varphi_2)\right)^{\frac{1}{2}}}\frac{W_3}{\left(1-k\,\mathrm{Sin}^2(\varphi_3)\right)^{\frac{1}{2}}}\right) \qquad (27)$$

It is notable that according to Table 1, for $ng=3$, $\xi_1=0.77459$, $\xi_2=-0.77459$, $\xi_3=0$, $W_1=0.55555$, $W_2=0.55555$ and $W_3=0.88888$. So by putting $L=0.5$ in (20), $H=0.5$ in (21) and $H_1=1$ in (22), the complex integrals in these equations reduce to some simple algebraic formulas including radicals. For example, equation (20) is transformed into the following equation:

$$L=\frac{1}{1296}C\pi^2\left(\frac{5\left(\frac{5}{\sqrt{1-0.0300522\alpha-0.938934\beta}}+\frac{8}{\sqrt{1-0.015507\alpha-0.484493\beta}}+\frac{5}{\sqrt{1-0.000961872\alpha-0.0300522\beta}}\right)}{\sqrt{1-0.0310141\alpha-0.968986\beta}}\right.$$

$$+\frac{8\left(\frac{5}{\sqrt{1-0.484493\alpha-0.484493\beta}}+\frac{8}{\sqrt{1-0.25\alpha-0.25\beta}}+\frac{5}{\sqrt{1-0.015507\alpha-0.015507\beta}}\right)}{\sqrt{1-0.5\alpha-0.5\beta}}$$

$$\left.+\frac{5\left(\frac{5}{\sqrt{1-0.938934\alpha-0.0300522\beta}}+\frac{8}{\sqrt{1-0.484493\alpha-0.015507\beta}}+\frac{5}{\sqrt{1-0.0300522\,\alpha-0.000961872\beta}}\right)}{\sqrt{1-0.968986\alpha-0.0310141\,\beta}}\right) \qquad (28)$$

after solving the system of equations (20)-(22), $\alpha=0.47$, $\beta=0.53$ and $C=0.12$ are obtained for this case. Now the height of the seepage face can be calculated by using these values, equation (3) and Gauss integral method by the following relationship:

$$H_0 = C\int_0^{\frac{\pi}{2}} \frac{K\left(\alpha \operatorname{Cos}^2(\varphi)\right)\operatorname{Sin}(\varphi)\operatorname{Cos}(\varphi)}{\left((1-\alpha_1 \operatorname{Sin}^2(\varphi))(1-\beta_1 \operatorname{Sin}^2(\varphi))\right)^{\frac{1}{2}}} d\varphi = \frac{\pi}{4} C\left(W_1 f_3(\varphi_1) + W_2 f_3(\varphi_2) + W_3 f_3(\varphi_3)\right) \tag{29}$$

Where, $f_3(\varphi_i)$ is defined in (26) and φ_i and ξ_i are the same as the ones implemented in equations (23)-(27) (ones assigned for $ng = 3$ in table 1).According to calculations in (28), the value of H_0 for $L = 0.5$ and $H = 0.5$ equals 0.16.

Tables(2)-(7) show the resulted H_0 obtained in this article for some other values of L and H for $ng = 3$ and $ng = 4$ .Also the related results ofHornung and Krueger (1985)to which our results have been compared, can be seen in these tables.

Table 2- Height of seepage face, H_0 ,in terms of the depth of the right lake, H ($L = 0.5$)

H	[14]	Present study (ng=3)	Error (%)	Present study (ng=4)	Error (%)
0.5	0.1624	0.1618	-0.37	0.1626	0.12
0.4	0.2471	0.2462	-0.36	0.2468	-0.12
0.3	0.3388	0.3376	-0.35	0.3383	-0.15
0.2	0.4344	0.4335	-0.21	0.4339	-0.12
0.1	0.5324	0.5323	-0.02	0.5320	-0.08
0	0.6318	0.6310	-0.13	0.6312	-0.09

Table 3- Height of seepage face, H_0 ,in terms of the depth of the right lake, H ($L = 0.6$)

H	[14]	Present study (ng=3)	Error (%)	Present study (ng=4)	Error (%)
0.5	0.1208	0.1198	-0.83	0.1209	0.08
0.4	0.1952	0.1948	-0.2	0.1950	-0.10
0.3	0.2796	0.2788	-0.35	0.2792	-0.14
0.2	0.3706	0.3698	-0.21	0.3700	-0.16
0.1	0.4659	0.4653	0.13	0.4655	-0.09
0	0.5645	0.5637	-0.13	0.5640	-0.09

Table 4- Height of seepage face, H_0 ,in terms of the depth of the right lake, H ($L = 0.7$)

H	[14]	Present study (ng=3)	Error (%)	Present study (ng=4)	Error (%)
0.5	0.0893	0.0857	-4.03	0.0893	0.00
0.4	0.1531	0.1520	-0.72	0.1532	0.07
0.3	0.2293	0.2285	-0.35	0.2291	-0.09
0.2	0.3145	0.3136	-0.29	0.3140	-0.16
0.1	0.4065	0.4065	0.00	0.4060	-0.12
0	0.5040	0.5032	-0.16	0.5035	-0.10

Table 5- Height of seepage face, H_0 ,in terms of the depth of the right lake, H ($L = 0.8$)

H	[14]	Present study (ng=3)	Error (%)	Present study (ng=4)	Error (%)
0.5	0.0660	0.0608	-7.88	0.0643	-2.58
0.4	0.1200	0.1165	-2.92	0.1199	-0.08
0.3	0.1877	0.1861	-0.85	0.1877	0.00
0.2	0.2668	0.2657	-0.41	0.2665	-0.11
0.1	0.3551	0.3550	-0.03	0.3547	-0.11
0	0.4513	0.4508	-0.11	0.4508	-0.11

Table 6- Height of seepage face, H_0 ,in terms of the depth of the right lake, H ($L=0.9$)

H	[14]	Present study (ng=3)	Error (%)	Present study (ng=4)	Error (%)
0.5	0.0488	0.0468	-4.10	0.0451	-7.58
0.4	0.0941	0.0892	-5.21	0.0926	-1.59
0.3	0.1540	0.1510	-1.95	0.1536	-0.26
0.2	0.2270	0.2255	-0.66	0.2266	-0.18
0.1	0.3114	0.3115	0.03	0.3108	-0.19
0	0.4063	0.4065	0.05	0.4061	-0.05

Table 7- Height of seepage face, H_0 ,in terms of the depth of the right lake, H ($L=1$)

H	[14]	Present study (ng=3)	Error (%)	Present study (ng=4)	Error (%)
0.5	0.0362	0.0398	9.94	0.0335	-7.46
0.4	0.0741	0.0715	-3.51	0.0705	-4.86
0.3	0.1268	0.1237	-2.44	0.1250	-1.42
0.2	0.1940	0.1926	-0.72	0.1932	-0.41
0.1	0.2746	0.2752	0.22	0.2742	-0.15
0	0.3682	0.3694	0.33	0.3682	0.00

According to tables (2)-(7), in most cases, there is a very good agreement between the results of this research and Hornung and Krueger (1985)and also it is evident that increasing Gauss points often decreases the errors and increases the accuracy of the results.SoGauss quadrature integral methodis a reliable method which facilitates computation of corresponding integrals greatly and can be used in analytical solution of seepage problem in porous media to minimize the computational time and effort.

For making the results more tangible, a 3D plot of H_0 as a function of L and H , has been sketched out inFigure 2.

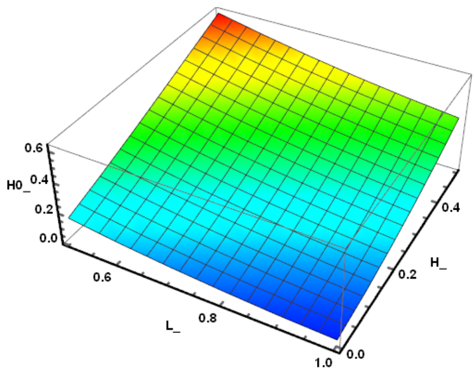

Figure 2. 3D plot of H_0 versus L and H

5. CONCLUSIONS

In this article, we considered Polubarinova-Kochina'sformulas whichare presented as an exact analytical solution to seepage problem through porous media. Gaussquadratureintegral method was utilized to compute the complicated integrals in these formulas.The complex formulas were transformed into some algebraic simple forms and even more simpler forms can be achieved in the future work. The results were compared to those of Hornung and Krueger (1985) and an acceptable agreement,which can be enhanced by increasing Gauss key points, was observed. So Gauss integral methodcan be consideredas an effective and fast way to determine the exact solution of a seepage problem whose results can be used to evaluate the accuracy and effectiveness of numerical methods.

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