Comparison of Ideal-Coupled and Decoupled Methods in Dynamic Analysis of Gravity Dams

Mohammad Rezaiee-Pajand[1], Mohammad Sadegh Kazemiyan[2], Zahra Mirjalili[1]
1- Department of Civil Engineering, Faculty of Engineering, Ferdowsi University of Mashhad, Iran
2- Toossab Consulting Engineers Company, Mashhad, Iran

Email:sadegh_kazemiyan@yahoo.com

Abstract
This paper aims to take advantage of the decoupled[1] and ideal-coupled[2] method in dynamic analysis of gravity dams. In this way, the capability of these methods is compared. The analysis is conducted in frequency domain, and the responses are transformed into the time domain with the help of Fourier transform. Note that, the ideal-coupled approach has not been applied in the analysis of gravity dams. The finite element formulation of dam-reservoir systems results in unsymmetric eigenvalue problem, when pressure and displacements are the water and the dam unknowns, correspondingly. It should be highlighted that the unsymmetriceigensolvers are usually very time-consuming from execution point of view, and also complicated from programming point of view. The previously mentioned schemes were developed to symmetrize this eigenvalue problem required to be solved in frequency-domain analysis.

Keywords: Concrete gravity dam, Decoupled method, Ideal-coupled method, Dynamic analysis.

1. INTRODUCTION

By using the finite element approach, the dynamic behavior of concrete gravity dam-reservoir systems can be studied. In usual, the dynamic analysis may be conducted in time or frequency domains. This type of analysis may be carried out in these domains either by direct or modal method. Therefore, finding the natural frequencies and corresponding mode shapes of the gravity dam is required in the dynamic analysis. To calculate natural frequencies and mode shapes, the eigen-value problem governing the free vibration of the dam-reservoir system should be solved.

For proposing a new symmetric version of the originally non-symmetric coupled eigen-problem governing the free vibration of fluid-structure systems, Sandberg [3] utilized the eigen-vectors of each domain. In this method, the displacement finite element formulation for the solid and either pressure or displacement potential for the fluid were employed. This strategy was the advent of developing new generation of symmetrizing schemes without requiring the coupled modes shapes. Similarly, Lotfi[1] took advantage of the decoupled mode shapes instead of the coupled ones in the modal analysis. In this method, the decoupled mode shapes were envisaged as the Ritz vectors. It should be reminded that the decoupled eigen-problems are symmetric. Then, some researchers compared the capability of the decoupled tactic with the coupled one [4]. Then, AftabiSani and Lotfi[2] applied new mode shapes entitled ideal-coupled modes in the modal analysis of concrete arch dams. These modes were employed in a similar manner to the decoupled modes. However, they were actually coupled mode shapes of two ideal fictitious systems. It is worthwhile to mention that the coupled eigen-problem of these systems were symmetric. Recently, Rezaiee-Pajand et al. developed a new strategy entitled "quadratic ideal-coupled method" for finding the eigenpairs of the arch dams[5].

In this paper, the decoupled and ideal-coupled method are employed for dynamic analysis of Pine Flat gravity dam. It should be reminded that the analysis is conducted in the frequency domain, and the results are transformed into the time domain. Then, their capabilities are compared. Note that, these methods have not been compared, yet.

2. ANALYSIS METHOD

To discretize the dam and fluid domain, the FE-(FE-HE) method is used. For brevity, the formulation is initially presented without considering the reservoir far-field region. Afterwards, the effects of this region areexplained for the general case. The coupled governing equation of the system has the coming appearance [4]:

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{B} & \mathbf{G} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{r}} \\ \ddot{\mathbf{p}} \end{bmatrix} + \begin{bmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{L} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{r}} \\ \dot{\mathbf{p}} \end{bmatrix} + \begin{bmatrix} \mathbf{K} & -\mathbf{B}^{\mathrm{T}} \\ \mathbf{0} & \mathbf{H} \end{bmatrix} \begin{bmatrix} \mathbf{r} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} -\mathbf{MJa}_{\mathrm{g}} \\ -\mathbf{BJa}_{\mathrm{g}} \end{bmatrix} \tag{1}$$

Where $\mathbf{M}$, $\mathbf{K}$ and $\mathbf{C}$ denote the mass, stiffness and damping matrix of the dam body, correspondingly. Moreover $\mathbf{G}$, $\mathbf{H}$ and $\mathbf{L}$ are the generalized mass, stiffness and damping of the fluid domain, respectively. And, $\mathbf{B}$ is the interaction matrix which relates the fluid pressure to the structural acceleration[6]. Additionally, vectors $\mathbf{r}$ and $\mathbf{p}$ contain the unknown nodal displacements and pressures, respectively. Furthermore, $\mathbf{J}$ is a matrix with each its two rows are an 2×2 identity matrix.Also, $\mathbf{a}_{\mathrm{g}}$ is the vector of ground accelerations. By conducting the Fourier transform, the matrix Eq. (1) can be rewritten into the next form:

$$\begin{bmatrix} -\omega^2\mathbf{M} + \mathbf{K}(1+2\beta_d \mathrm{i}) & -\mathbf{B}^{\mathrm{T}} \\ -\omega^2\mathbf{B} & -\omega^2\mathbf{G} + \mathrm{i}\omega\mathbf{L} + \mathbf{H} \end{bmatrix} \begin{bmatrix} \mathbf{r} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} -\mathbf{MJa}_{\mathrm{g}} \\ -\mathbf{BJa}_{\mathrm{g}} \end{bmatrix} \tag{2}$$

where i and ω are the imaginary unit and the natural frequency of the system, respectively. It should be reminded that the hysteretic damping matrix is used in the aforementioned relation. This matrix has the subsequent shape[2]:

$$\mathbf{C} = \frac{2\beta_d}{\omega}\mathbf{K} \tag{3}$$

In the last relation, β_d is the constant hysteretic factor of the dam body. It is worth emphasizing that the coupled equation of a dam with the finite reservoir system in the frequency domain is shown by Eq. (2).

3. FREE VIBRATION ANALYSIS

It is clear that the eigenvalue problem corresponding to Eq. (2) can be written as follows [6]:

$$\left(\omega^2 \begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{B} & \mathbf{G} \end{bmatrix} + \begin{bmatrix} -\mathbf{K} & \mathbf{B}^{\mathrm{T}} \\ \mathbf{0} & -\mathbf{H} \end{bmatrix}\right) \begin{bmatrix} \mathbf{r} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} \tag{4}$$

It is clear that the above-mentioned linear eigenvalue problem is similar to that of the free vibration equation of un-damped systems. But it is not symmetric. This unsymmetrical linear eigenvalue problem is required to be solved for finding the eigenpairs of the dam-reservoir system. The actual coupled eigenpairs can be calculated by directly solving the original eigenvalue problem (4). Using the obtained eigenvectors in the modal analysis results in more accurate responses, in comparison to the other available alternatives. Nevertheless, the standard eigen-solvers cannot be employed for solving this equation because of its unsymmetry. According to other researchers, the unsymmetrical eigenvalue solution routines are more time-consuming than symmetrical ones. From the programming point of view, these methodsare more complicated, as well[2, 4, 7]. In this work, the decoupled and ideal-coupled method are applied.

4. DECOUPLED EIGENPROBLEM

By removing the interaction matrix $\mathbf{B}$, a symmetric shape of the original eigenvalue problem (4) can be obtained[1]:

$$\left(\omega^2 \begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{G} \end{bmatrix} - \begin{bmatrix} \mathbf{K} & \mathbf{0} \\ \mathbf{0} & \mathbf{H} \end{bmatrix}\right) \begin{bmatrix} \mathbf{r} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} \tag{5}$$

Obviously, this eigenvalue problem is symmetric. Hence, it can be solved by using standard eigen-solvers. The eigenvectors of these symmetric equations are not the true mode shapes of the actual system, but they can be utilized in a modal analysis method named "decoupled modal strategy". It should be noted that the decoupled eigenvectors can be considered as the Ritz vectors. Consequently, it can be shown that employing all of these modes leads to the exact answers. It is worthwhile to mention that the eigenvalues achieved from the decoupled eigenproblem are the natural frequencies of the dam and reservoir separately[2].

5. IDEAL-COUPLED EIGENPROBLEM

Herein, instead of the true coupled one, the eigenproblems corresponding to two ideal dam-reservoir systems are solved. In the first ideal system, the fluid is assumed to be incompressible, and the dam is massless in the second one. In comparison to the decoupled ones, the eigenvalues of these problems are closer to the natural frequencies of the actual coupled dam-reservoir system. Moreover, the obtained eigenvectors are more similar to the actual ones. These vectors can be used in a modal analysis approach named "ideal-coupled modal strategy" [2].The simplified form of the first ideal eigenproblem has the following shape:

$$\left[\omega^2\left(\mathbf{M}+\mathbf{M}_{\mathrm{a}}\right)-\mathbf{K}\right]\mathbf{r}=\mathbf{0} \tag{6}$$

In this equation, $\mathbf{M}_{\mathrm{a}}$ denotes the added mass matrix and can be calculated as follows:

$$\mathbf{M}_{\mathrm{a}}=\mathbf{B}^{\mathrm{T}}\mathbf{H}^{-1}\mathbf{B} \tag{7}$$

The pressure vector can be obtained by employing the succeeding equation:

$$\mathbf{p}=\omega^2\mathbf{H}^{-1}\mathbf{B}\,\mathbf{r} \tag{8}$$

Clearly, the size of this eigenproblem is equal to the number of the unknown nodal displacements. The second ideal eigenvalue problem has the next appearance:

$$\left[\omega^2\left(\mathbf{G}+\mathbf{G}_{\mathrm{a}}\right)-\mathbf{H}\right]\mathbf{p}=\mathbf{0} \tag{9}$$

where

$$\mathbf{G}_{\mathrm{a}}=\mathbf{B}\mathbf{K}^{-1}\mathbf{B}^{\mathrm{T}} \tag{10}$$

The displacement vector can be obtained by utilizing the next relation:

$$\mathbf{r}=\mathbf{K}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{p} \tag{11}$$

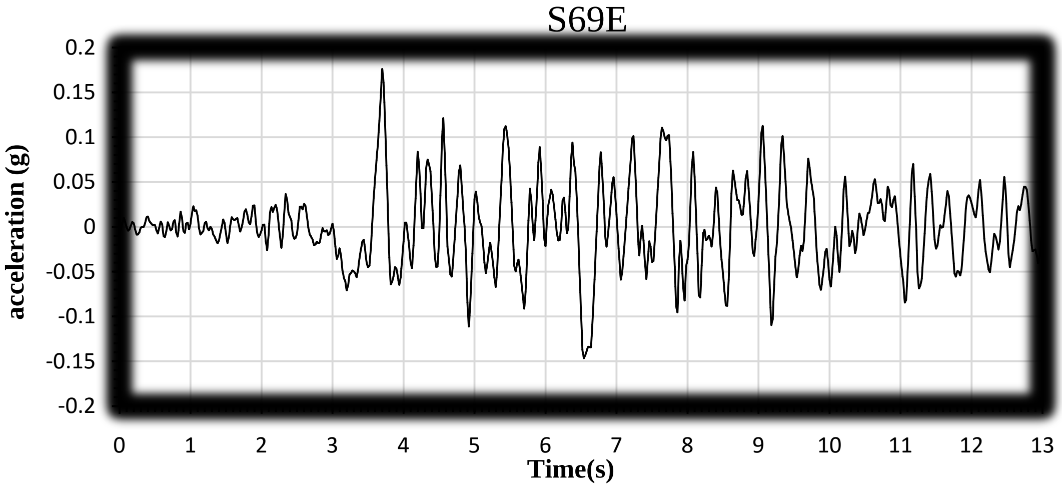
It is obvious that the size of the second ideal eigenvalue problem equals the number of unknown nodal pressures of the fluid domain.The above-cited ideal eigenproblems can be rewritten as below:

$$\left(\omega^2\begin{bmatrix}\mathbf{M}+\mathbf{M}_{\mathrm{a}} & \mathbf{0}\\ \mathbf{0} & \mathbf{G}+\mathbf{G}_{\mathrm{a}}\end{bmatrix}-\begin{bmatrix}\mathbf{K} & \mathbf{0}\\ \mathbf{0} & \mathbf{H}\end{bmatrix}\right)\begin{bmatrix}\mathbf{r}\\ \mathbf{P}\end{bmatrix}=\begin{bmatrix}\mathbf{0}\\ \mathbf{0}\end{bmatrix} \tag{12}$$

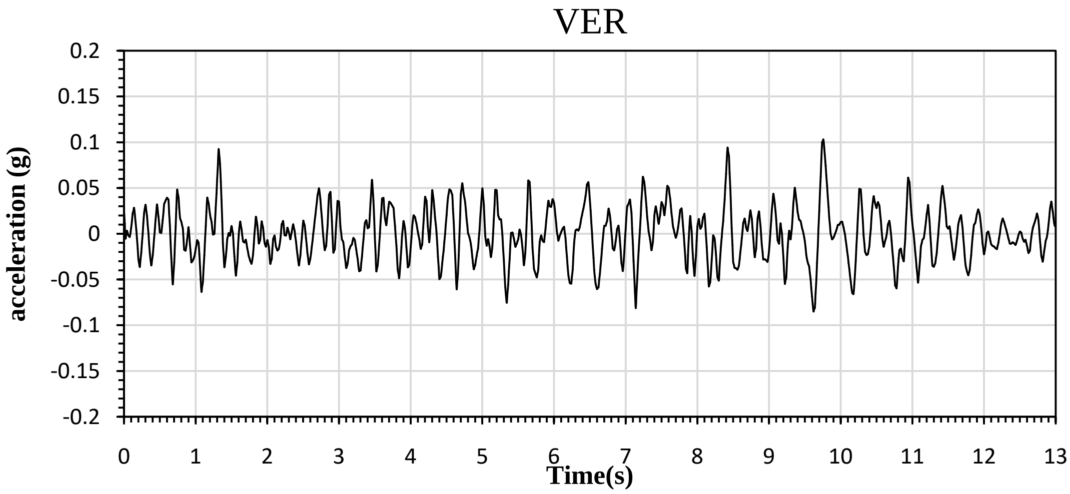
This eigenproblem is linear and symmetric. Hence, its solution can be calculated by employing the standard common methods. It is clear that omitting $\mathbf{M}_{\mathrm{a}}$ and $\mathbf{G}_{\mathrm{a}}$ from Eqs. (6)-(9) leads to the decoupled eigenvalue problem. Therefore, the decoupled form of the actual eigenproblem is a special case of the ideal-coupled one. It should be remarked that the ideal-coupled approach is more accurate than the decoupled one[2].

6. NUMERICAL EXAMPLES

In this work, the finite element method was used for the main part of the analysis procedure. For this purpose, a computer program was developed based on the theories proposed in the previous sections. Accordingly, the solid finite elements are applied for modeling the dam, and the near-field and far-field fluid domains are discretized by the fluid finite elements and hyper-elements[8], correspondingly. It should be mentioned that the true coupled, decoupled and ideal-coupled are the available options for the dynamic modal analysis of the gravity dams in this computer program. In the subsequent sections, to comparethese methods, they are employed for the dynamic analysis of Pine Flat gravity dam under vertical and horizontal components of Taft earthquake [9]. In Figures 1, the ground motion record corresponding to this earthquake are presented.



(a)



(b)

Figure 1. Taft earthquake component

The responses of the dam crests are computed as a result of horizontal and vertical excitations for reflection coefficient (α) equal 1. Note that; $\alpha = 1$ represents the full reflection. The achieved results are compared with the exact ones (i.e. those achieved from the direct method using all the true coupled mode shapes).

6.1. MODELING

Herein, the aforesaid methods are applied in the dynamic analysis of Pine Flat gravity dam. The finite element model of the dam on a rigid foundation is studied. The dam is discretized by 40 isoparametric 8-node plane-solid finite elements. It should be mentioned that the water domain contains near-field and far-field regions. After the near-field region, the far-field part starts and extends to infinity in the upstream direction. 90 isoparametric 8-node plane-fluid elements are used for modeling the near-field region with the length of 200 m, and the far-field part is modeled by a fluid hyper-element, including 9 isoparametric 3-node sub-elements. The used mesh pattern was previously applied by other researchers [4]. In Figure 2, the finite element model of Pine Flat dam and its reservoir are shown.

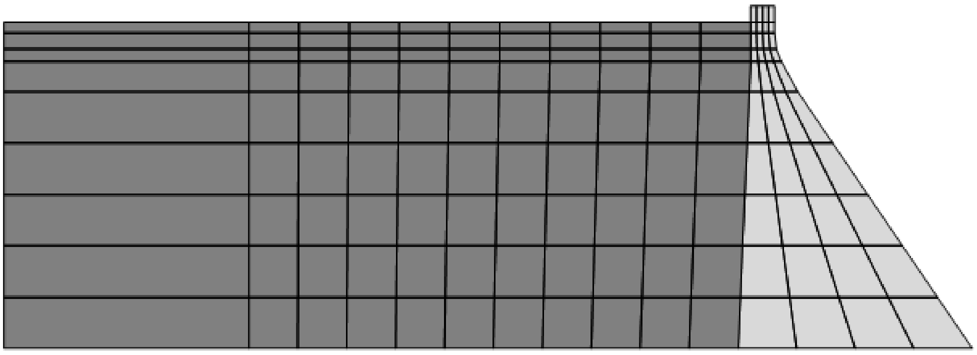


Figure 2. Dam body with the near-field and far-field fluid regions

It should be added that Pine Flat dam has a sloped upstream face. The hyper-elements should connect to the vertical sides of the finite elements to have more accuracies[8, 9]. As a result, this slope should gradually be diminished prior to connecting the hyper-elements to the finite elements in the finite element region.The dam body is made of homogenous concrete with isotropic linearly viscoelastic behavior whose elasticity modulus, Poisson's ratio and unit weight are $22.75\,\mathrm{Gpa}$, 0.2 and $24.8\ \mathrm{kN/m^3}$, respectively. Moreover, the hysteretic damping factor is assumed to be 0.05. Additionally, the impounded water is presumed to be compressible, inviscid and irrotational, and its unit weight and pressure wave velocity are $9.81\,\mathrm{kN/m^3}$ and $1440\,\mathrm{m/s}$, correspondingly.

6.2. FREE VIBRATION RESPONSES

At the first stage, the first five natural frequencies of this dam are presented in Tables 1 and 2.

Table 1- The first five natural frequencies of Pine Flat dam-reservoir system with $L = 200m$

Mode Number	Natural frequencies f_i (Hz)		
	Decoupled [4]	Ideal-coupled	True coupled [4]
	Dam	First ideal case (incompressible fluid assumption)	
1	3.15	2.67	2.53
2	6.48	5.77	3.27
3	8.74	8.66	4.67
4	11.25	10.35	6.22
5	16.99	15.98	7.92

Table 2- The second five natural frequencies of Pine Flat dam-reservoir system with $L = 200m$

Mode Number	Natural frequencies f_i (Hz)		
	Decoupled [4]	Ideal-coupled	True coupled [4]
	Reservoir	Second ideal case (incompressible fluid assumption)	
1	3.12	2.94	2.53
2	4.75	4.24	3.27
3	7.80	6.05	4.67
4	9.30	7.92	6.22
5	9.96	9.46	7.92

It is clear that the ideal-coupled results are more accurate, in comparison to those of the decoupled one.

6.3. TIME HISTORY RESPONSES

At this stage, the crest responses of PineFlat dam are presented in time domain for 13 sec. It should be mentioned that the effect of static loads including hydrostatic pressure and the dam weight are ignored. As previously mentioned, the dynamic analysis is conducted in the frequency domain and the responses are transformed into the time domain by using Fourier Transform. Herein, two modes of the dam-reservoir system are applied. It should be reminded that the decoupled, ideal coupled and true coupled eigenpairs are used in this process, and the obtained results are compared. In Figures3, the displacement of the dam crest under the horizontal component of Taft earthquake are presented, respectively.

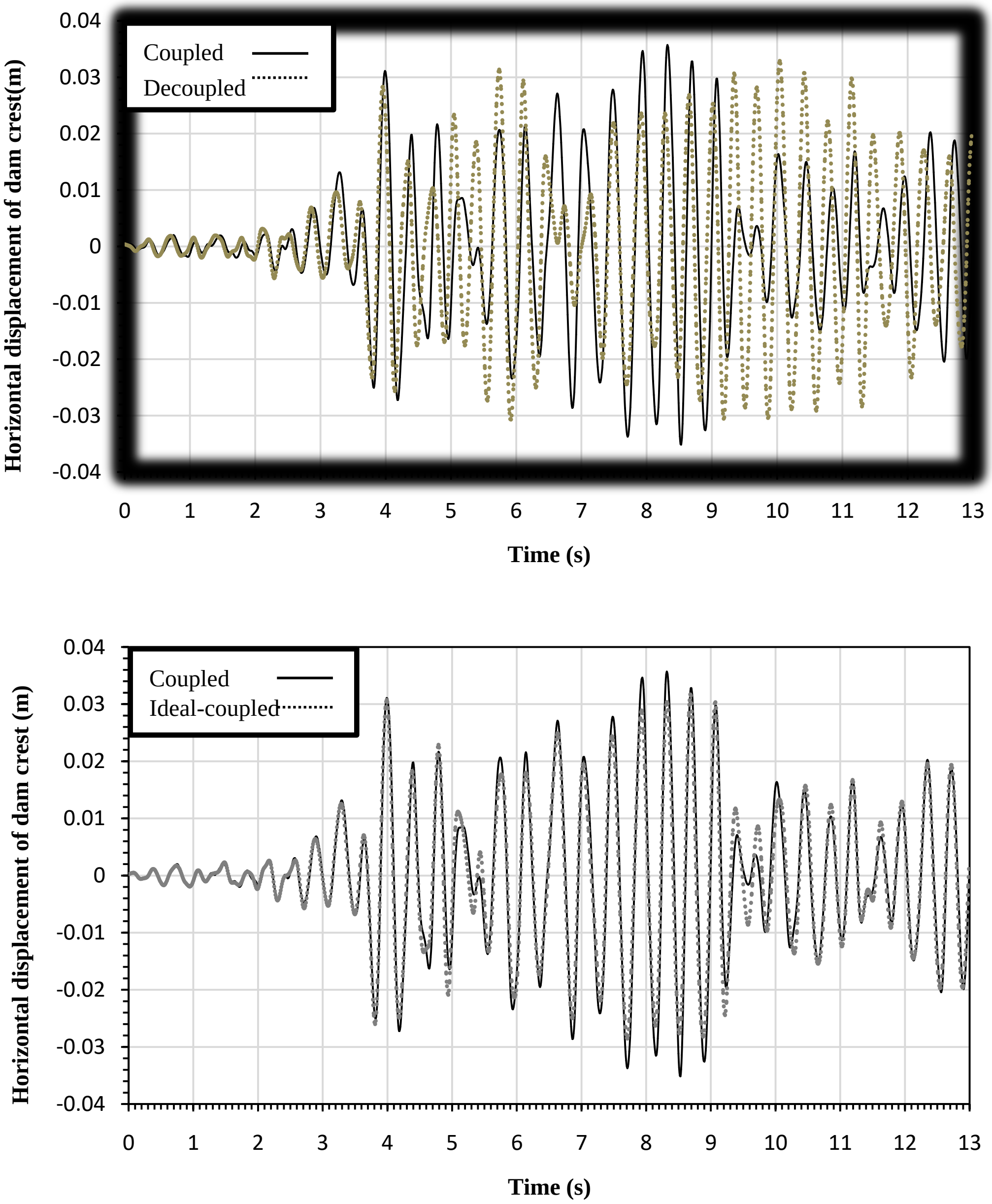


Figure 3. Time history of dam crest based on horizontal excitation for $\alpha = 1$

In Figures4, the crest displacement of PineFlat dam is presented under vertical component of Taft earthquake for $\alpha = 1$.

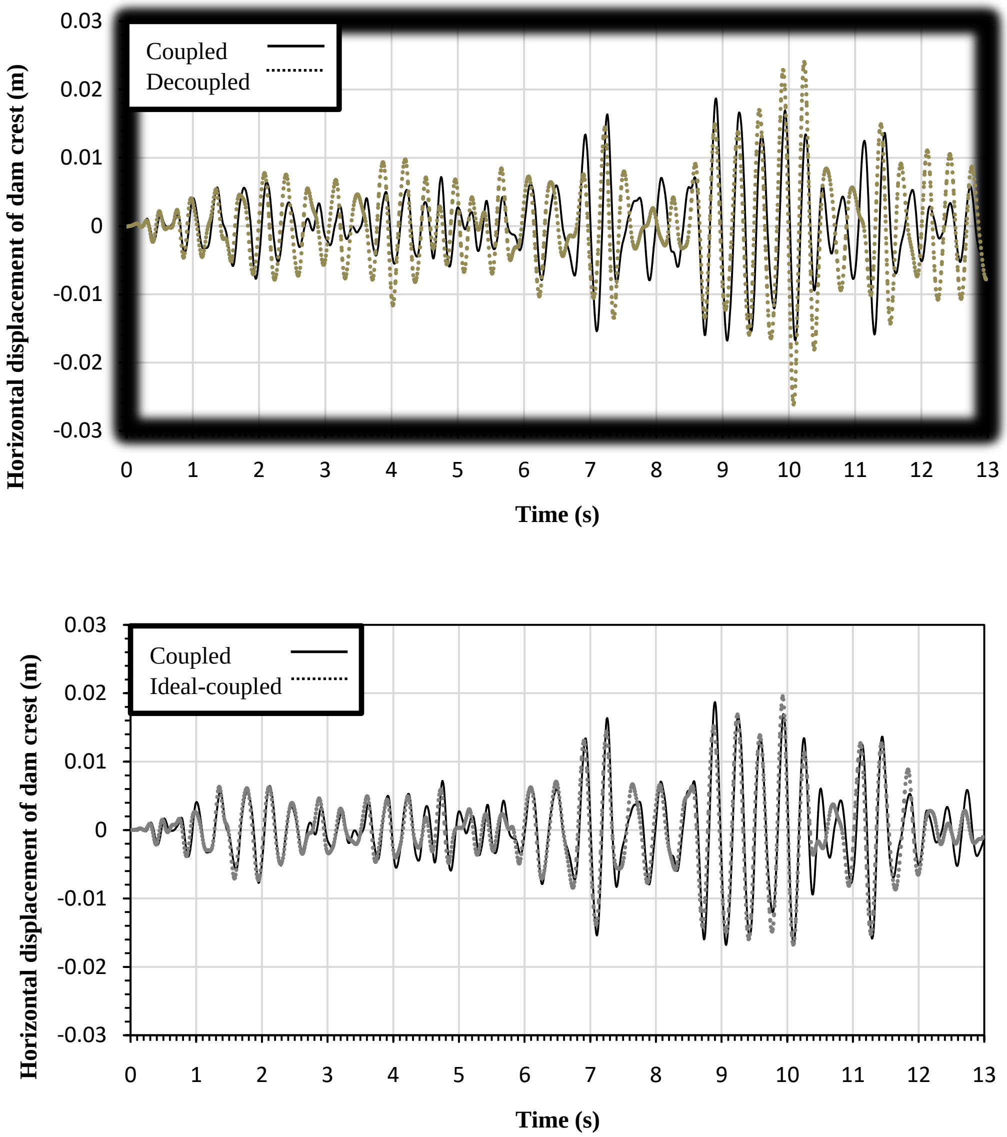


Figure 4. Time history of dam crest based on vertical excitation for $\alpha = 1$

It is obvious that, by using the ideal-coupled method in modal dynamic analysis more accurate results can be achieved, in comparison to the decoupled approach.

7. CONCLUSIONS

In usual modal dynamic analysis is conducted for assessing the dynamic behavior of concrete gravity dams. For this purpose, the eigen-value problem governing the free vibration of this system is required to be solved. For this purpose, various options, including true coupled, decoupled and ideal-coupled methods, are available. The first one takes advantage of the accurate mode shapes while the others use the approximate ones. This paper was devoted to employ the decoupled and ideal-coupled approaches in modal dynamic analysis of PineFlat gravity dam to compare the accuracy of these approaches. Findings shows that using ideal-coupled approach leads to more accurate responses, in comparison to the decoupled one.

8. REFERENCES

1. Lotfi, V.(2005),“Frequency domain analysis of concrete arch dams by decoupled modal approach,”Structural Engineering and Mechanics, 21(4), p. 423-435.
2. Aftabi S, A. and Lotfi, V.(2010),“Dynamic analysis of concrete arch dams by ideal-coupled modal approach,” Engineering Structures, 32(5), p. 1377-1383.
3. Sandberg, G.(1995),“A new strategy for solving fluid-structure problems,”International Journal for Numerical Methods in Engineering, 38,p. 357-370.
4. Samii, A. and Lotfi, V.(2007),“Comparison of coupled and decoupled modal approaches in seismic analysis of concrete gravity dams in time domain,”Finite Elements in Analysis and Design,43, p. 1003-1012.
5. Rezaiee-Pajand, M., Aftabi S, A. and Kazemiyan ,M.S.(2018),“Free vibration analysis of concrete arch dams by quadratic ideal-coupled method,”Structural Engineering and Mechanics, 65(1), p. 69-79.
6. Rezaiee-Pajand, M., Aftabi S, A. and Kazemiyan ,M.S.(2016),“Analytical solution for free vibration of flexible 2D rectangular tanks,”Ocean Engineering, 122, p. 118-135.
7. Arjmandi, S.A. and Lotfi, V.(2011), “Computing mode shapes of fluid-structure systems using subspace iteration methods,”Scientia Iranica A, 18(6), p. 1159-1169.
8. Hojati, M. and Lotfi, V.(2011), “Dynamic analysis of concrete gravity dams utilizing two-dimensional modified efficient fluid hyper-element,”Advances in Structural Engineering, 14(6), p. 1093-1106.
9. Aftabi S, A. and Lotfi, V.(2011),“ An effective procedure for seismic analysis of arch dams including dam-reservoir-foundation interaction effects,”Journal of Earthquake Engineering, 15(7), p. 971-988.